Location modelling for community healthcare facilities

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Overview

• Application to community healthcare
• Classical location models
• Models
  - hierarchical
  - efficiency/ equity
• Output
Application

Community health services
• rural districts of developing countries
• rural/metropolitan Leeds
Hierarchical healthcare facilities

Key
vw village worker
cc community centre
h hospital
Hierarchical systems

Highest level most specialised services, fewest facilities

- Single-flow
  Entry at lowest level only

- Multi-flow
  Entry at multiple levels
  - Successively-inclusive
    Services available at higher levels
  - Exclusive
    Services available only at particular levels
Classical location models

• $p$-Median - essential services
  Hakimi (1964, 1965)
  – Minimise total population-weighted distance travelled to the nearest facility

• Maximal covering - limited cover services
  Church and Revelle (1974)
  – Maximise total population within cover distance/time of a facility

*The number of facilities to be located is specified.*
Equity objectives

- Minimise total absolute deviation from desirable service standard
  - \( p \)-Median Distance
  - Maximum cover Population per facility
Hierarchical models

$p$-Median
- HiMIn-PMP-Eq
- HiMEx-PMP-Eq
- HiS-PMP-Eq

Max Cover
- HiMIn-MCL-Eq
- HiMEx-MCL-Eq
- HiS-MCL-Eq
HiMIn-PMP-Eq

decision variables

\[ X^k_j = \begin{cases} 
1 & \text{if a level } k \text{ facility is located at node } j, \\
0 & \text{otherwise,} 
\end{cases} \quad j \in J, k = 1, 2, \ldots, K. \]

\[ Y_{ij} = \begin{cases} 
1 & \text{if demand at node } i \text{ is allocated to a facility at node } j, \\
0 & \text{otherwise,} 
\end{cases} \quad i \in I, j \in J. \]
Minimise

\[ Z = \alpha \sum_{i \in I} \sum_{j \in J} d_{ij} p_i r_i Y_{ij} + (1 - \alpha) \sum_{i \in I} \sum_{j \in J} p_i r_i Y_{ij} \left| d_{ij} - S_A \right| \]
HiMIn-PMP-Eq

constraints

subject to

Minimum distance;
All demand satisfied uniquely;
Allocation to an open facility

\[ X_j^k + Y_{il} \leq 1, \quad i \in I, j \in J, k \in K, l \in J \mid d_{ij} < d_{il} \]

\[ \sum_{j \in J} Y_{ij} = 1, \quad i \in I, \]

\[ Y_{ij} \leq \sum_{k=1}^{K} X_j^k, \quad i \in I, j \in J. \]
Common constraints

Within referral distance between levels;
Number of facilities per level;
Pre-existing facilities

\[ X_j^k \leq \sum_{l \in J} r_{jl}^{k} X_l^{k+1}, \quad j \in J, k = 1,2,\ldots,K - 1, \]

\[ \sum_{j \in J} X_j^k = N_k, \quad k = 1,2,\ldots,K, \]

\[ X_j^k \geq p_j^k, \quad j \in J, k = 1,2,\ldots,K. \]
HiMIn-MCL-Eq

Decision variables

\[ X_j^k = \begin{cases} 1 & \text{if a level } k \text{ facility is located at node } j, \quad j \in J, k = 1, 2, \ldots, K. \\ 0 & \text{otherwise,} \end{cases} \]

\[ Y_{ij}^k = \begin{cases} 1 & \text{if demand at node } i \text{ is allocated to a level } k \text{ facility at node } j, \\ 0 & \text{otherwise,} \end{cases} \quad i \in I, j \in J, k = 1, 2, \ldots, K. \]

\[ Z_{1,j}^k, Z_{2,j}^k \quad \text{used in calculating absolute values} \quad j \in J, k = 1, 2, \ldots, K. \]
HiMIn-MCL-Eq

Objective

Maximise

\[ Z = \alpha \sum_{i \in I} \sum_{j \in J} \sum_{k=1}^{K} p_i r_i Y_{ij}^k - (1 - \alpha) \sum_{j \in J} \sum_{k=1}^{K} (Z_{1,j}^k + Z_{2,j}^k) \]

subject to

\[ Z_{1,j}^k - Z_{2,j}^k = \sum_{i \in I} p_i r_i Y_{ij}^k - S_A, \quad j \in J, k = 1, 2, \ldots, K, \]

absolute deviation from service standard

efficiency

weight

equity

covered by facility \( j \)

desirable service standard
HiMIn-MCL-Eq
Max cover at any level – constraints on allocation

Demand allocated to nearest open facility at any level;
Demand can be allocated at some level only if covered at that level;
Can allocate demand once only;
Demand must be allocated if covered by an open facility.

\[
c_{ij}^k X_j^k + \sum_{q=1}^{K} Y_{il}^q \leq 1, \quad i \in I, j \in J, k = 1,2,...,K, l \in J \mid d_{ij} \leq d_{il},
\]
\[
Y_{ij}^k \leq c_{ij}^k X_j^k, \quad i \in I, j \in J, k = 1,2,...,K
\]
\[
\sum_{j \in J} \sum_{k=1}^{K} Y_{ij}^k \leq 1, \quad i \in I.
\]
\[
\sum_{i \in I} \sum_{c=1}^{K} Y_{ij}^c \geq X_j^k, \quad i \in I, j \in J, k = 1,2,...,K \mid c_{ij}^k > 0.
\]
Output from HiMEx-PMP-Eq, equity 0 and 1
Output from HiMEx-PMP-Eq, location of 3 low level and 1 high level facilities.
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