Location modelling for community healthcare facilities

Mrs. Honora K. Smith, Dr. Paul R. Harper,
Prof. Chris N. Potts,
OR Group, School of Mathematics,
University of Southampton

Overview

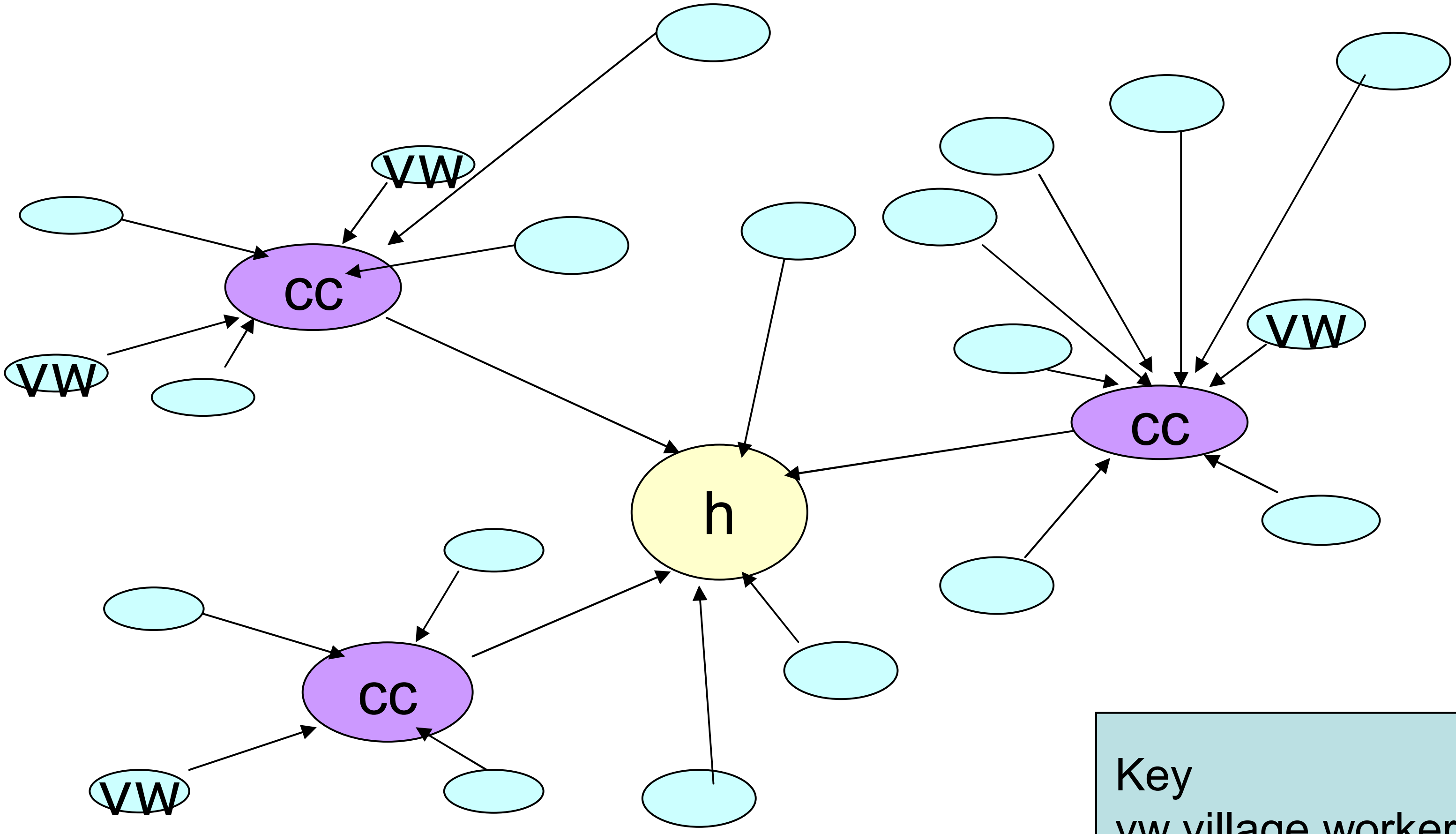
- Application to community healthcare
- Classical location models
- Models
    - hierarchical
    - efficiency/ equity
- Output

Application

Community health services

- rural districts of developing countries
- rural/metropolitan Leeds

Hierarchical healthcare facilities



Key
vw village worker
cc community centre
h hospital

Hierarchical systems

Highest level most specialised services, fewest facilities

- Single-flow

  Entry at lowest level only

- Multi-flow

  Entry at multiple levels

  - Successively-inclusive

    Services available at higher levels

  - Exclusive

    Services available only at particular levels

Classical location models

- *p*-Median - essential services

  Hakimi (1964, 1965)

  - Minimise total population-weighted distance travelled to the nearest facility
- Maximal covering - limited cover services

  Church and Revelle (1974)

  - Maximise total population within cover distance/time of a facility

The number of facilities to be located is specified.

Equity objectives

- Minimise total absolute deviation from desirable service standard
  - *p*-Median

    Distance
  - Maximum cover

    Population per facility

Hierarchical models

p-Median

- ❖HiMIn-PMP-Eq
- ❖HiMEx-PMP-Eq
- ❖HiS-PMP-Eq

Max Cover

- ❖HiMIn-MCL-Eq
- ❖HiMEx-MCL-Eq
- ❖HiS-MCL-Eq

HiMIn-PMP-Eq

decision variables

Location; allocation

$$X_j^k = \begin{cases} 1 & \text{if a level } k \text{ facility is located at node } j, \\ 0 & \text{otherwise,} \end{cases} \qquad j \in J, k = 1,2,\ldots,K.$$

$$Y_{ij} = \begin{cases} 1 & \text{if demand at node } i \text{ is allocated to a facility at node } j, \\ 0 & \text{otherwise,} \end{cases} \qquad i \in I, j \in J.$$

HiMIn-PMP-Eq

objective

distance weight

desirable service standard

Minimise

$$Z = \alpha \underbrace{\sum_{i \in I} \sum_{j \in J} d_{ij} p_i r_i Y_{ij}}_{\text{efficiency}} + (1-\alpha) \sum_{i \in I} \sum_{j \in J} \underbrace{p_i r_i Y_{ij} \, | d_{ij} - S_A |}_{\text{equity}}$$

HiMIn-PMP-Eq

constraints

subject to

Minimum distance;

All demand satisfied uniquely;

Allocation to an open facility

$$X_j^k + Y_{il} \le 1, \qquad i \in I, j \in J, k \in K, l \in J \mid d_{ij} < d_{il}$$

$$\sum_{j \in J} Y_{ij} = 1, \qquad i \in I,$$

$$Y_{ij} \le \sum_{k=1}^{K} X_j^k, \qquad i \in I, j \in J.$$

Common constraints

Within referral distance between levels;

Number of facilities per level;

Pre-existing facilities

$$X_j^k \leq \sum_{l \in J} r_{jl}^k X_l^{k+1}, \qquad j \in J, k = 1,2,\ldots,K-1,$$

$$\sum_{j \in J} X_j^k = N_k, \qquad k = 1,2,\ldots K,$$

$$X_j^k \geq p_j^k, \qquad j \in J, k = 1,2,\ldots,K.$$

HiMIn-MCL-Eq

Decision variables

$$X_j^k = \begin{cases} 1 & \text{if a level } k \text{ facility is located at node } j, \\ 0 & \text{otherwise,} \end{cases} \qquad j \in J, k = 1,2,\ldots,K.$$

$$Y_{ij}^k = \begin{cases} 1 & \text{if demand at node } i \text{ is allocated to a level } k \text{ facility at node } j, \\ 0 & \text{otherwise,} \end{cases}$$

$$i \in I, j \in J, k = 1,2,\ldots,K.$$

$Z_{1,j}{}^k, Z_{2,j}{}^k$ used in calculating absolute values $\qquad j \in J, k = 1,2,\ldots,K.$

HiMIn-MCL-Eq

Objective

absolute deviation from
service standard

weight

Maximise

$$Z = \alpha \underbrace{\sum_{i\in I}\sum_{j\in J}\sum_{k=1}^{K} p_i r_i Y_{ij}^k}_{\text{efficiency}} - (1-\alpha)\underbrace{\sum_{j\in J}\sum_{k=1}^{K}(Z_{1,j}^k + Z_{2,j}^k)}_{\text{equity}}$$

subject to

$$Z_{1,j}^k - Z_{2,j}^k = \underbrace{\sum_{i\in I} p_i r_i Y_{ij}^k}_{\text{covered by facility } j} - S_A, \qquad j \in J, k = 1,2,\ldots,K,$$

desirable service standard

HiMIn-MCL-Eq

Max cover at any level –constraints on allocation

Demand allocated to nearest open facility at any level;

Demand can be allocated at some level only if covered at that level;

Can allocate demand once only;

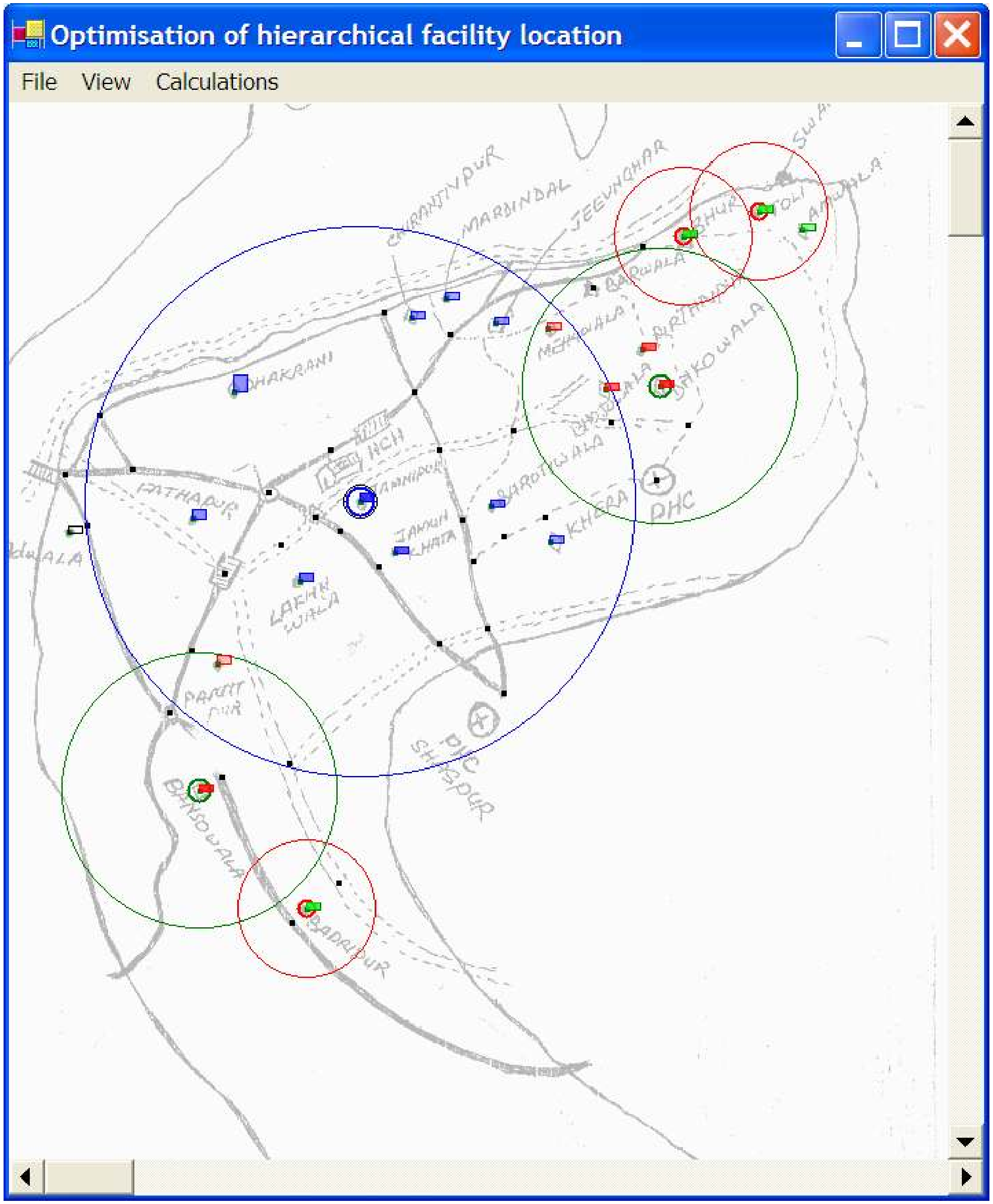
Demand must be allocated if covered by an open facility.

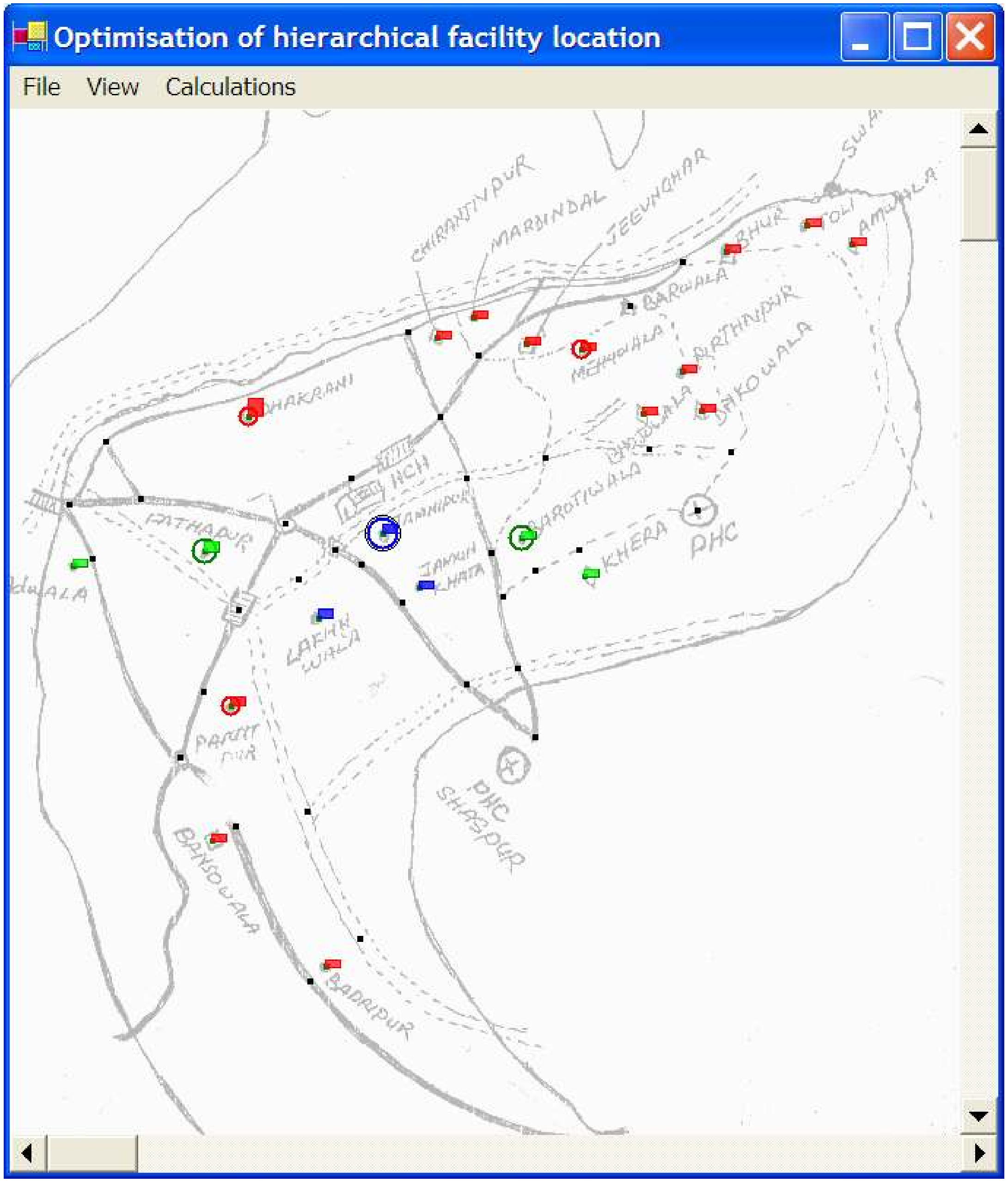
$$c_{ij}^k X_j^k + \sum_{q=1}^{K} Y_{il}^q \le 1, \qquad i \in I, j \in J, k = 1,2,\dots,K, l \in J \mid d_{ij} \le d_{il},$$

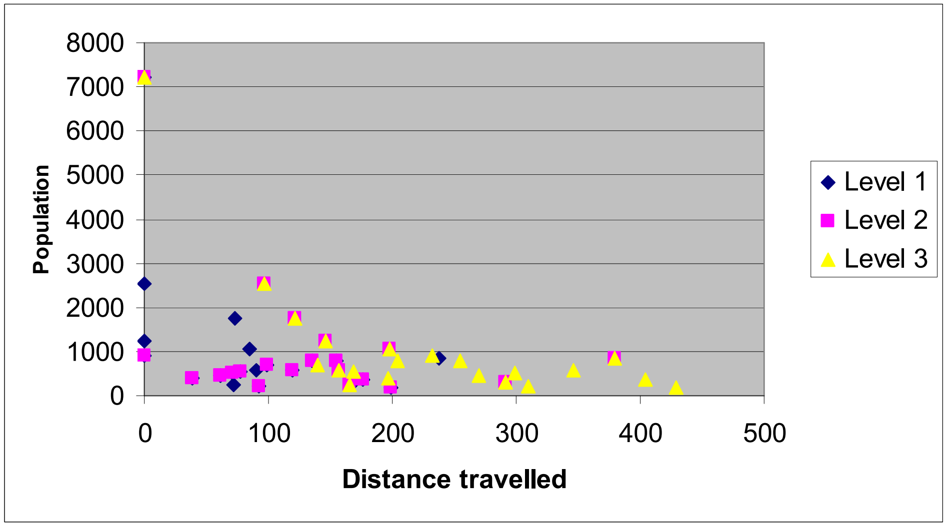
$$Y_{ij}^k \le c_{ij}^k X_j^k, \qquad i \in I, j \in J, k = 1,2,\dots,K$$

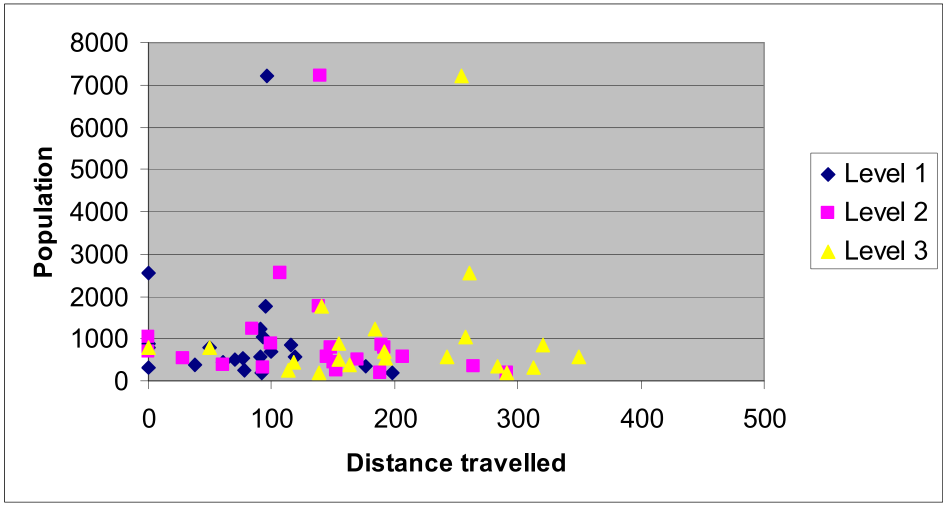
$$\sum_{j \in J} \sum_{k=1}^{K} Y_{ij}^k \le 1, \qquad i \in I.$$

$$\sum_{i \in I} \sum_{c=1}^{K} Y_{ij}^c \ge X_j^k, \qquad i \in I, j \in J, k = 1,2,\dots,K \mid c_{ij}^k > 0.$$

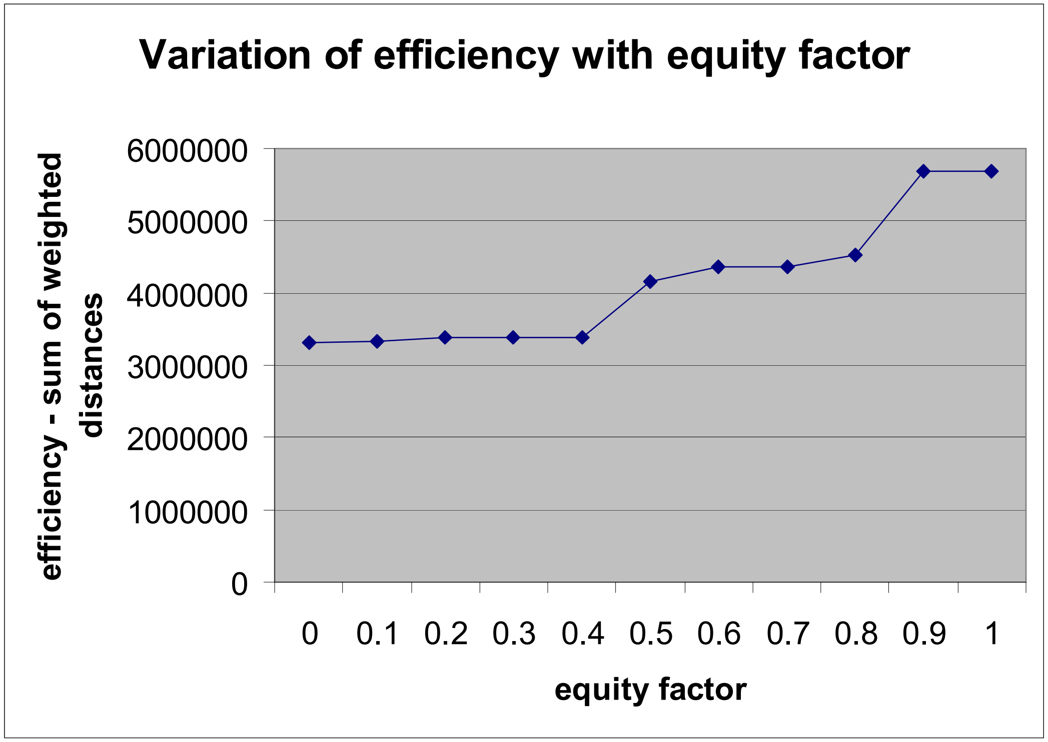

Optimisation of hierarchical facility location
File
View
Calculations
DHAKRANI
HCH
FATHAPUR
LAKHANWALA
KHERA
PHC
BARWALA
SAKOWALA
BANSOWALA
BADRIPUR
PHC
SHAHPUR


Optimisation of hierarchical facility location
File
View
Calculations
CHIRANJIVPUR
MARBINDAL
JEEVNGHAR
BHUR
TOLI
AMWALA
BARWALA
MEHUWALA
PIRTHIPUR
DAKOWALA
DHAKRANI
HCH
JAMNIPUR
PATHAPUR
BAROTIWALA
JAMUN KHATA
KHERA
PHC
LAKHAN WALA
PARTIT PUR
PHC SHASPUR
BANSOWALA
BADRIPUR

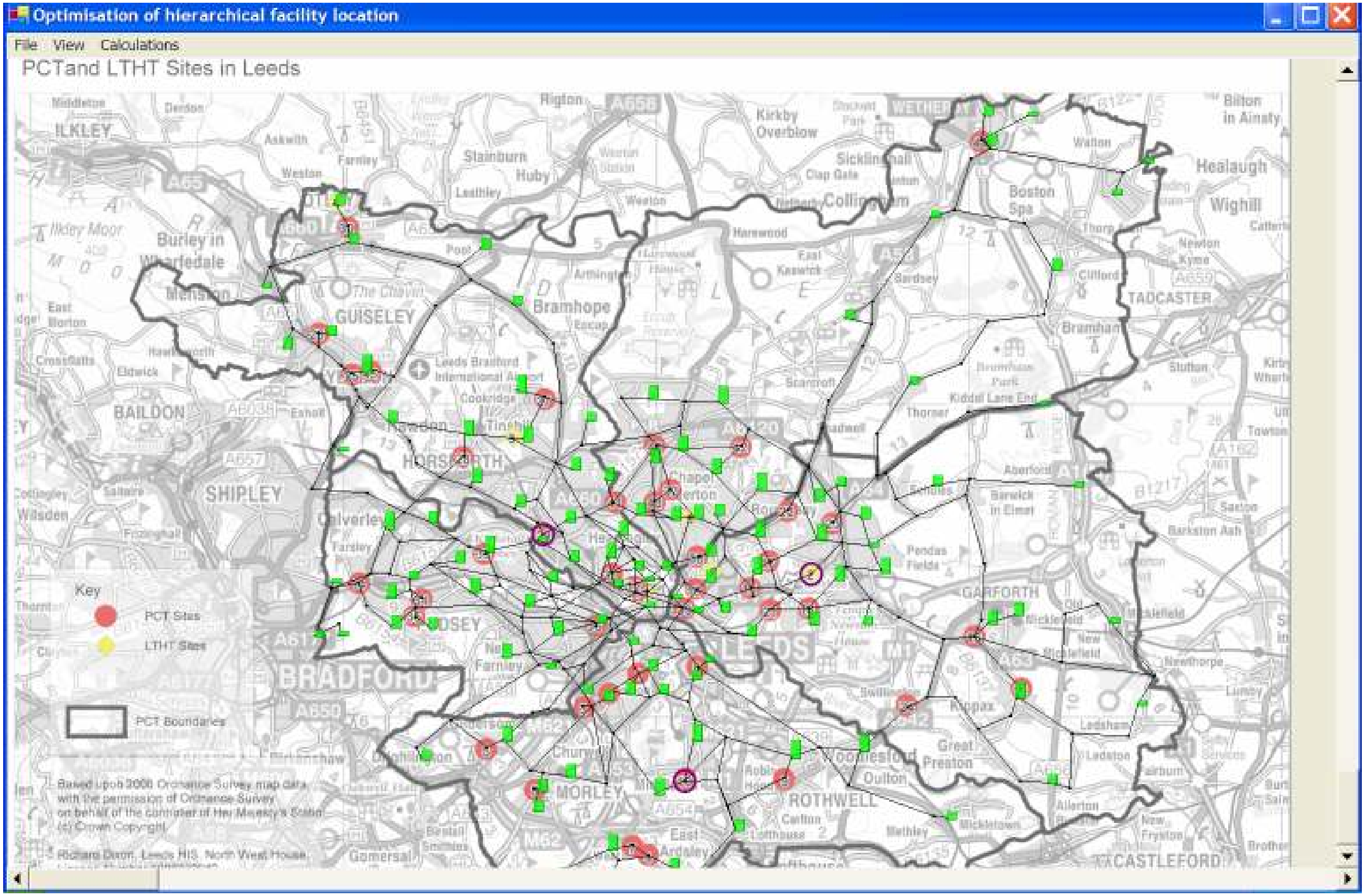




Output from HiMEx-PMP-Eq, equity 0 and 1



Output from HiMEx-PMP-Eq, location of 3 low level and 1 high level facilities.


Optimisation of hierarchical facility location
File
View
Calculations
PCTand LTHT Sites in Leeds
Key
PCT Sites
LTHT Sites
PCT Boundaries
Based upon 2006 Ordnance Survey map data
with the permission of Ordnance Survey
on behalf of the controller of Her Majesty's Stationery
(c) Crown Copyright
Richard Dixon, Leeds HIS, North West House
ILKLEY
Burley in Wharfedale
GUISELEY
BAILDON
SHIPLEY
BRADFORD
HORSFORTH
Bramhope
Leeds Bradford International Airport
Kirkby Overblow
Collingham
Boston Spa
Healaugh
Wighill
TADCASTER
Bramham
GARFORTH
Kippax
Great Preston
ROTHWELL
MORLEY
East Ardsley
Woodlesford
Oulton
Gomersal
CASTLEFORD
LEEDS
Stainburn
Huby
Harewood
Sicklinghall
Thorner
Scarcroft
Barwick in Elmet
Aberford
Calverley
Pudsey
Churwell
Ledsham
Ledston
Methley
Mickletown
Allerton Bywater
Swillington
Micklefield

Acknowledgements

- We are grateful for the assistance of
  - Emmanuel Hospital Association of North India;
  - Making Leeds Better.
- Xpress is used under an Academic Partnership with Dash Optimisation Ltd.
- Leeds maps are based on 2006 Ordnance Survey map data, with the permission of Ordnance Survey on behalf of the controller of Her Majesty's Stationery Office © Crown Copyright.
- This work has been funded by EPSRC, the Engineering and Physical Sciences Research Council of the UK.