Determining Shift Schedules for 24-hour Queuing Systems with Varying Arrival Rates

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College of Business Administration

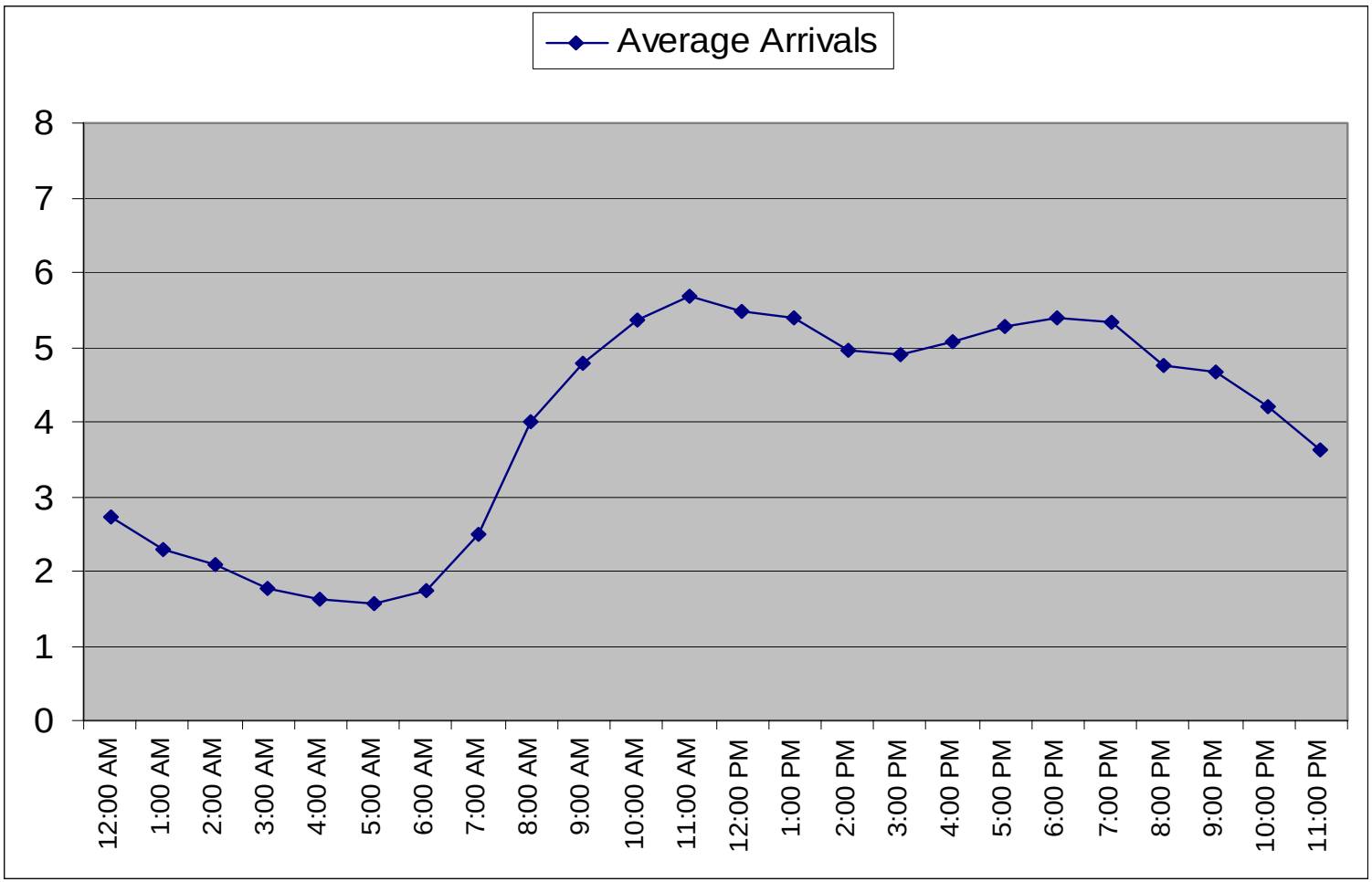
Overview

- The problem and its characteristics
- A two-step approach
  - Descriptive Simulation
  - Prescriptive Optimization
- Excel-based implementation
- Results & Observations

Example Problem - Situation

- Medium sized 24-hour Emergency Department with 95 visits per day, on average.

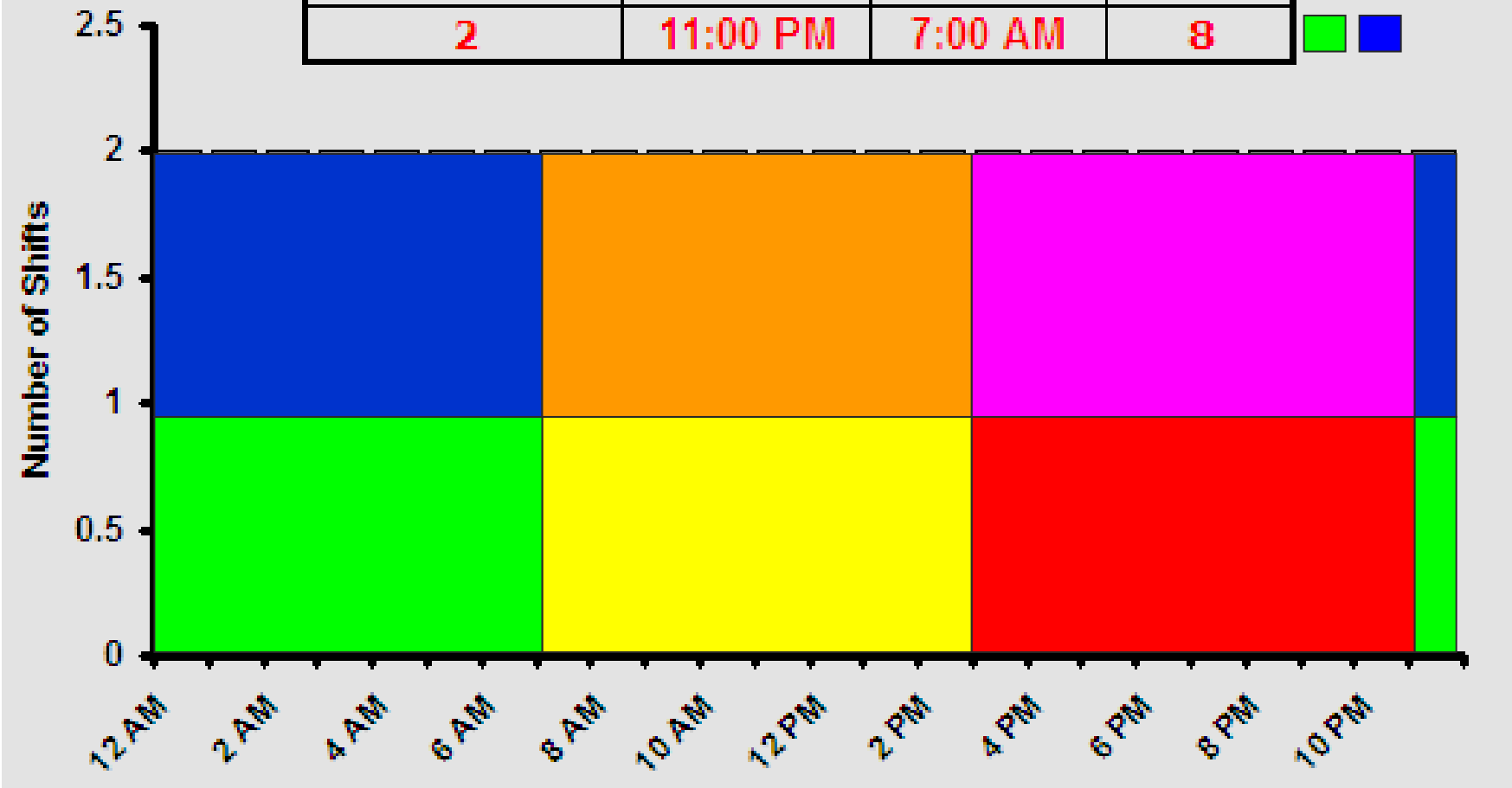
Example Problem - Situation



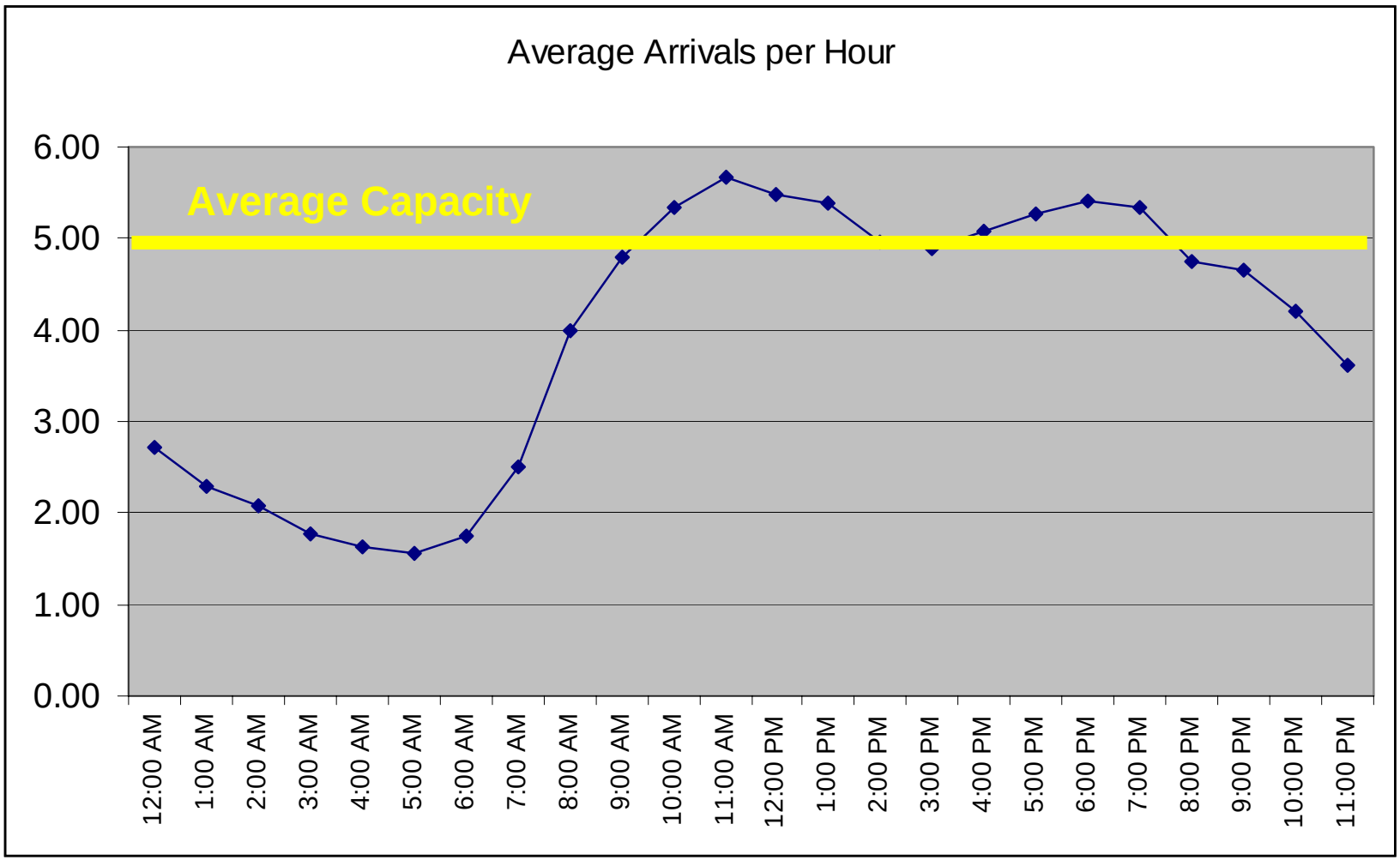
Example Problem - Situation

- Medium sized 24-hour Emergency Department with 95 visits per day, on average.
- Physicians can treat 2.5 patients per hour, on average. So, 38 (computed as 95/2.5) physician-hours required per day.
- Therefore, staffing 2 physicians on duty 24-hours per day should provide sufficient capacity cushion (estimated utilization of 79%, computed as 38/48).
- Furthermore, approximately 40 minute wait according to M/M2 estimate.

Number Of Shifts(Staff)	Start Time	End Time	Shift Length
2	7:00 AM	3:00 PM	8
2	3:00 PM	11:00 PM	8
2	11:00 PM	7:00 AM	8



Example Problem - Situation



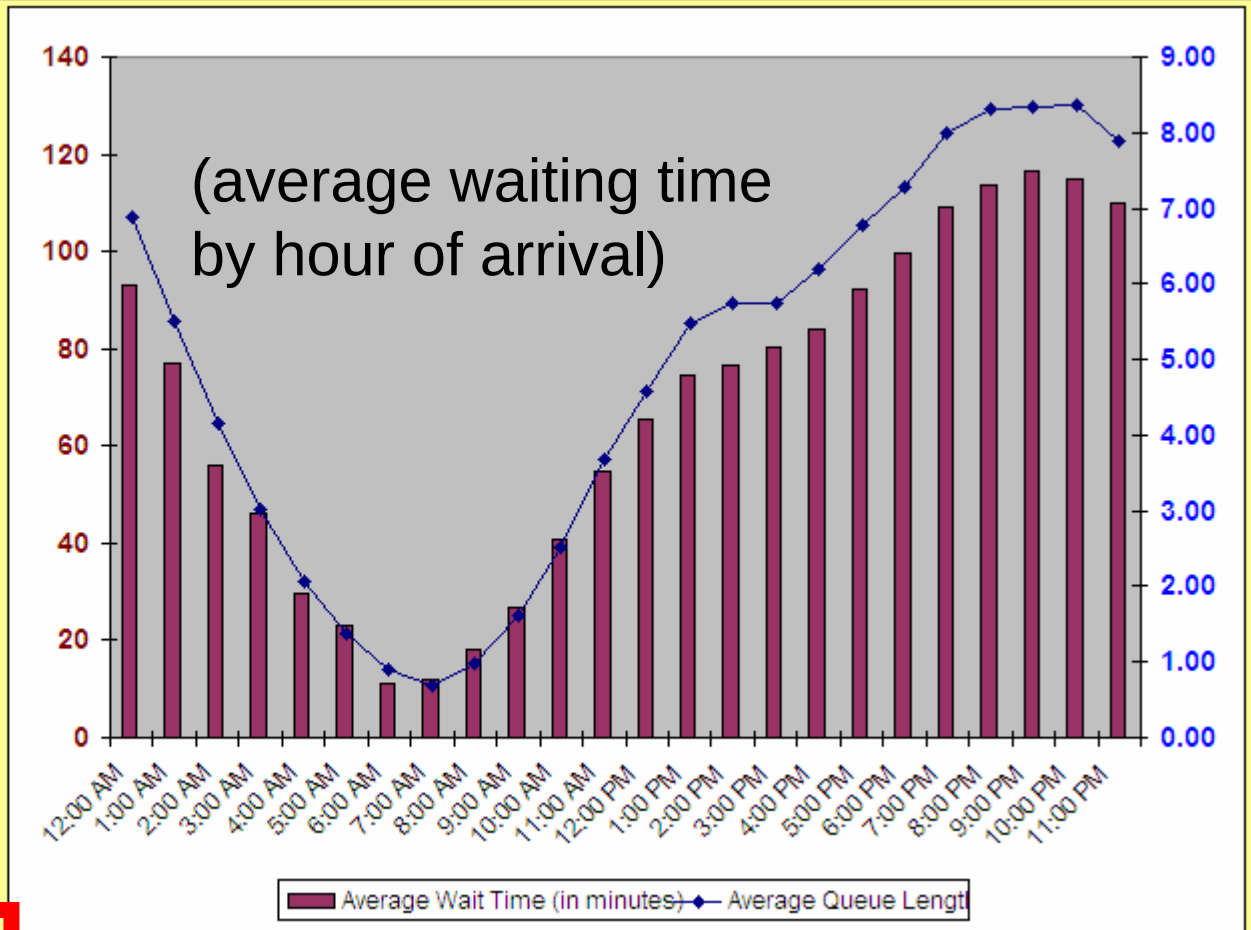
Example Problem

- Makes sense, but how will it really perform?
- Simulate for one year with assumed Poisson arrivals and Exponentially distributed service times.

Example Problem – Performance

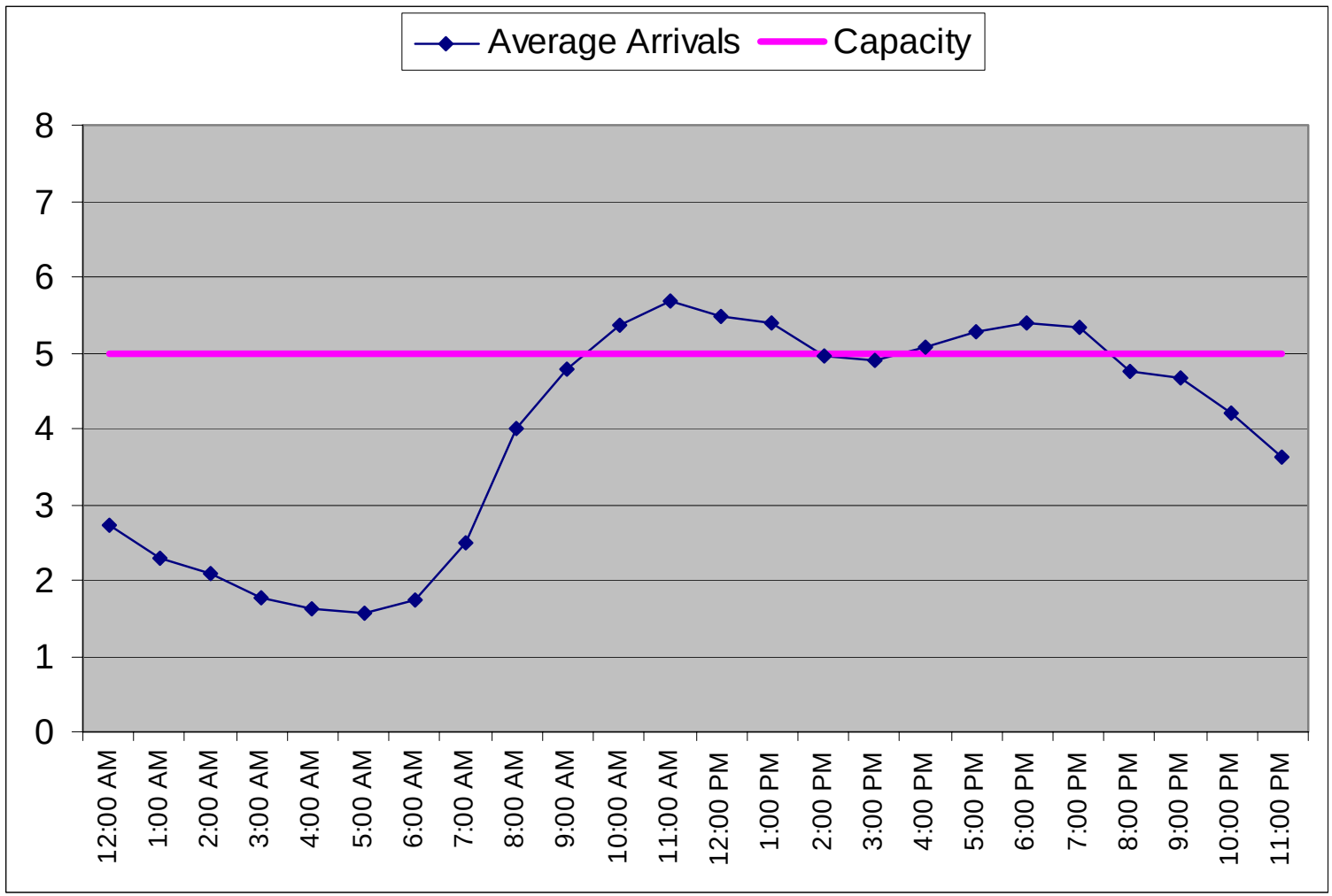
Arrival Hour	Average Number of Arrivals	Average Wait Time (in minutes)	Average Queue Length
12:00 AM	2.71	93	6.87
1:00 AM	2.23	77	5.52
2:00 AM	2.05	56	4.15
3:00 AM	1.66	46	3.02
4:00 AM	1.51	30	2.07
5:00 AM	1.56	23	1.38
6:00 AM	1.75	11	0.90
7:00 AM	2.53	12	0.70
8:00 AM	4.29	18	0.97
9:00 AM	4.89	27	1.62
10:00 AM	5.30	41	2.52
11:00 AM	5.73	55	3.68
12:00 PM	5.23	65	4.58
1:00 PM	5.41	74	5.48
2:00 PM	4.84	77	5.74
3:00 PM	4.74	80	5.74
4:00 PM	5.04	84	6.18
5:00 PM	5.27	92	6.78
6:00 PM	5.21	100	7.28
7:00 PM	5.45	109	7.99
8:00 PM	4.69	114	8.32
9:00 PM	4.67	116	8.33
10:00 PM	4.16	115	8.37
11:00 PM	3.66	110	7.90

Annual Number of Patients Served	
Average Wait (minutes) per Patient	73.76



Average patient wait is 74 minutes

Example Problem - Situation

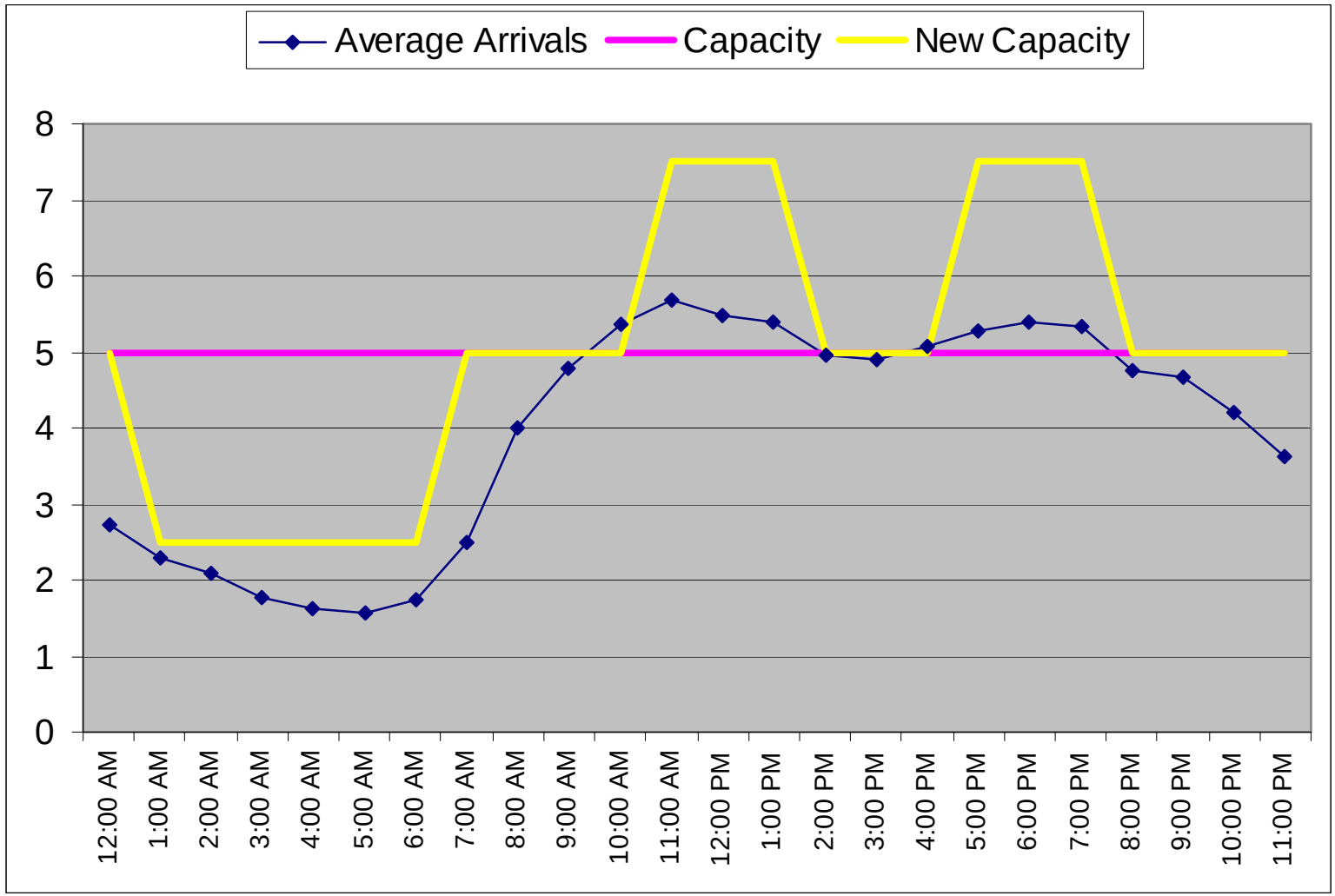


Example Problem - Challenges

- The average patient waiting is very bad and we know it can be improved by “better aligning” capacity with the demand.

- Two challenges:
  - What should the profile of capacity be?
  - What should be the shift schedule that supports it?

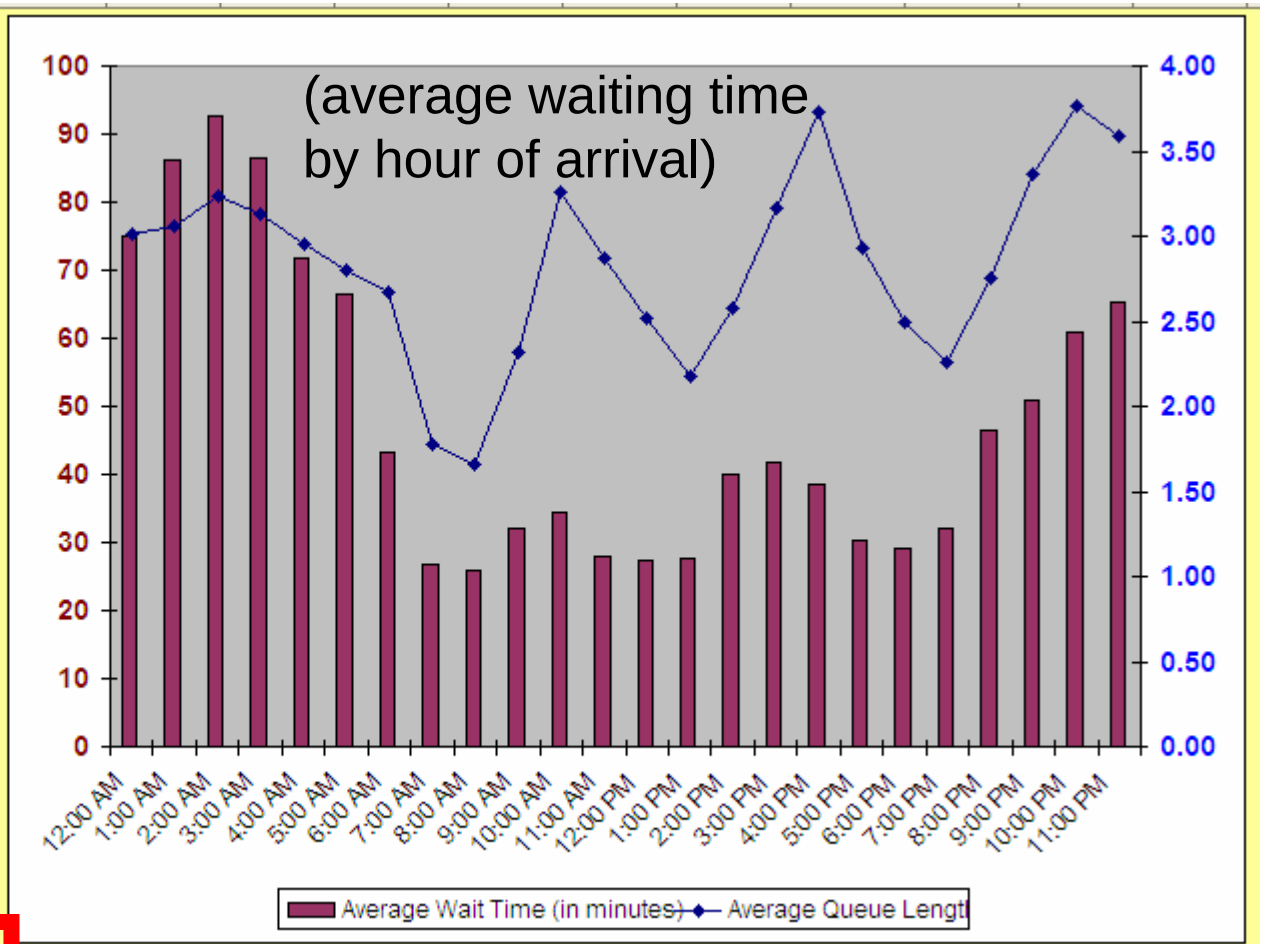
Example Problem - Resolution



Example Problem – Performance

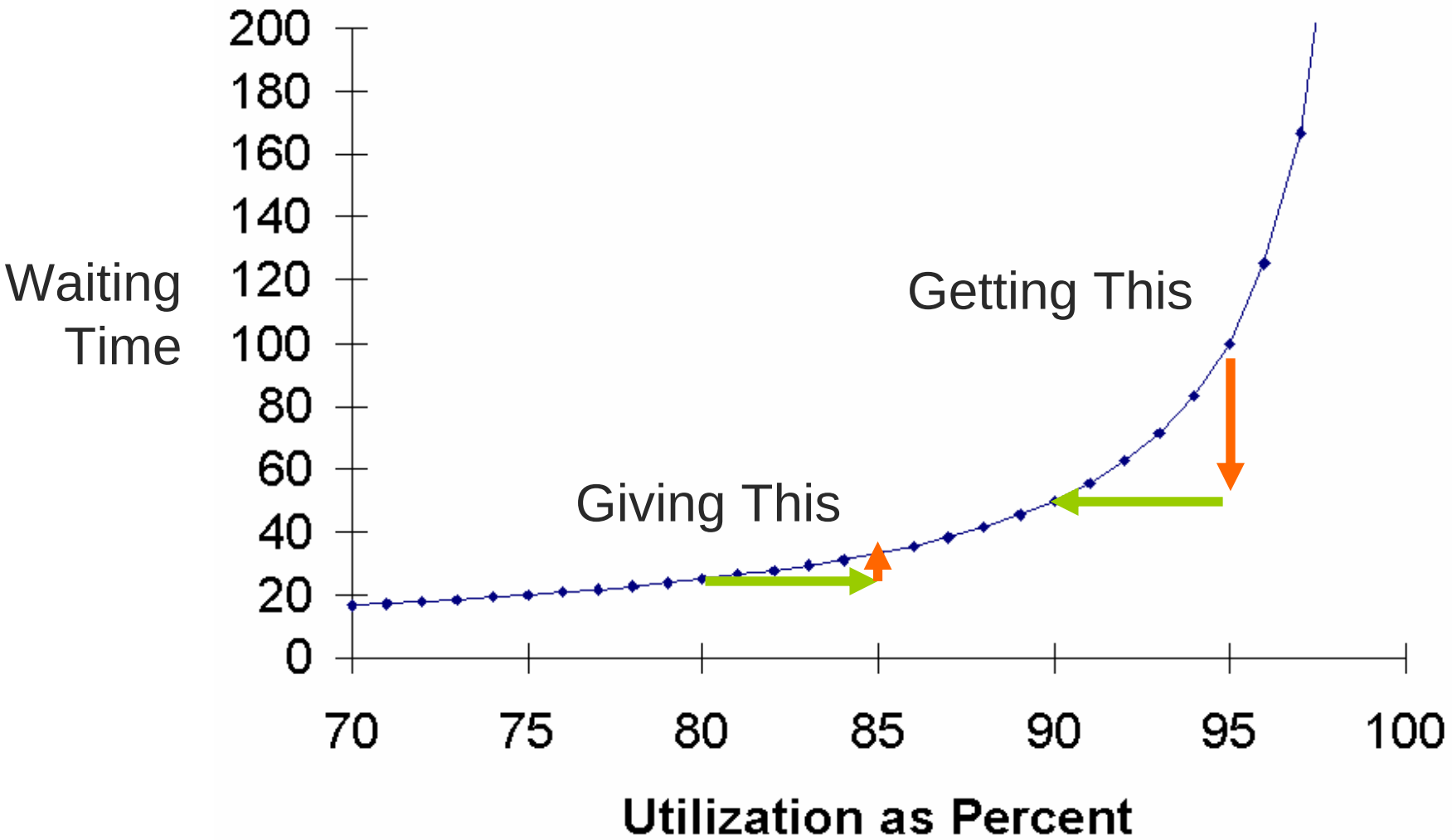
Arrival Hour	Average Number of Arrivals	Average Wait Time (in minutes)	Average Queue Length
12:00 AM	2.88	75	3.01
1:00 AM	2.23	86	3.05
2:00 AM	2.08	93	3.23
3:00 AM	1.77	86	3.12
4:00 AM	1.69	72	2.95
5:00 AM	1.62	66	2.80
6:00 AM	1.74	43	2.67
7:00 AM	2.41	27	1.77
8:00 AM	4.12	26	1.66
9:00 AM	5.03	32	2.32
10:00 AM	5.46	35	3.25
11:00 AM	5.76	28	2.87
12:00 PM	5.56	27	2.52
1:00 PM	5.11	28	2.17
2:00 PM	5.01	40	2.57
3:00 PM	4.74	42	3.16
4:00 PM	5.22	39	3.73
5:00 PM	5.18	30	2.93
6:00 PM	5.50	29	2.49
7:00 PM	5.27	32	2.26
8:00 PM	4.67	46	2.76
9:00 PM	4.74	51	3.37
10:00 PM	4.47	61	3.76
11:00 PM	3.57	65	3.58

Annual Number of Patients Served	[illegible]
Average Wait (minutes) per Patient	42.50

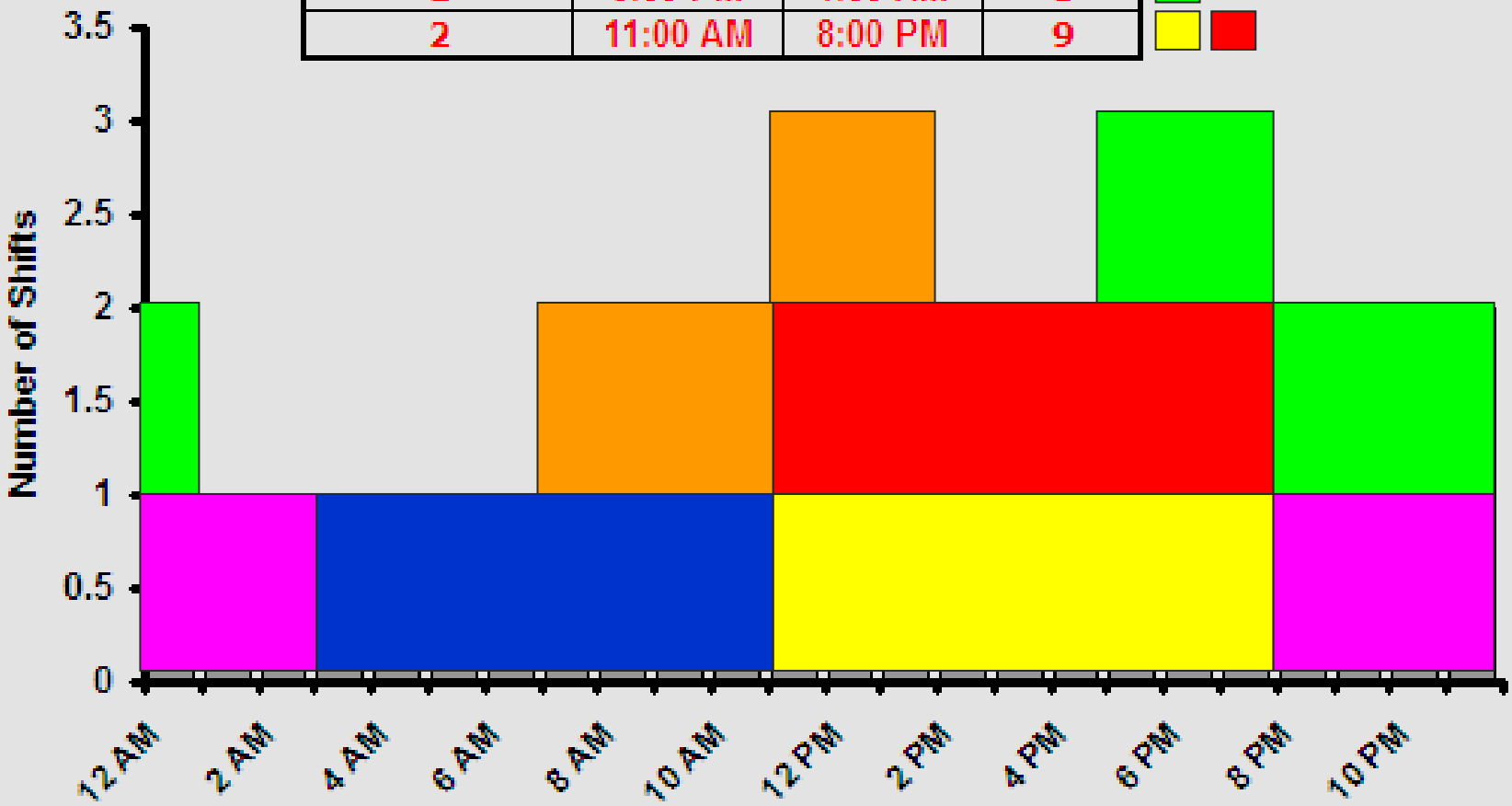


Average patient wait is 43 minutes

Typical Queue Behavior



Number Of Shifts(Staff)	Start Time	End Time	Shift Length
1	7:00 AM	2:00 PM	7
1	8:00 PM	3:00 AM	7
1	3:00 AM	11:00 AM	8
1	5:00 PM	1:00 AM	8
2	11:00 AM	8:00 PM	9



The Problem

Minimize $\sum_i \lambda_i [w_i(\lambda, \mu, p)]$

Subject to,

$A x = p$

$c x \leq H$

$x_j \geq 0$, integer for j=1,…J

$p_i \geq 0$, integer for i=1,…,24

Where λ and μ are vectors of arrival and service rates by hour and $a_{ij} = 1$ if candidate shift j covers hour i, and 0 otherwise.

J is the cardinality of the set of candidate shifts and H is the maximum number of server hours.

$w_i(\lambda, \mu, p)$ = the average waiting time for arrivals during period i.

Problem - Challenges

- The average patient waiting is very bad and we know it can be improved by “better aligning” capacity with the demand.
- Two challenges:
  - What should the profile of capacity be?
  - What should be the shift schedule that supports it?
- Goal is to reduce average patient waiting with no additional provider hours…

Problem - Challenges

- The average patient waiting is very bad and we know it can be improved by “better aligning” capacity with the demand.

- Two challenges:
  - What should the profile of capacity be?
  - What should be the shift schedule that supports it?

- Goal is to reduce average patient waiting with no additional provider hours… with ease.

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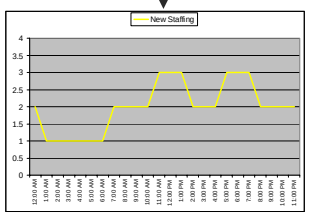
The Approach

Simulation to help determine staffing profile for 24-hours



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Minimize $\sum \lambda_i [w_i(\lambda, \mu, p')]$

Subject to,

$A x = p$

$c x \leq H$

$x_j \geq 0$, integer for j=1,…J

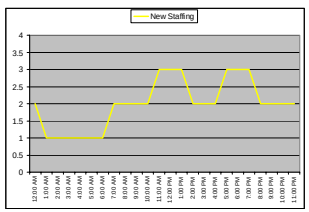
$p_i \geq 0$, integer for i=1,…,24

$\sum p'_i \leq H$

Descriptive Model - Advantages

- Relative to utilization-based methods, it accounts for non-linear behavior related to pooling.
- Relative to steady-state estimates (classic SIPP and Lag SIPP), it allows periods where average demand can exceed capacity.
- Relative to time-varying arrivals approximations, it can easily accommodate general service functions and multiple types of servers.
- It is descriptive and allows a user to explore alternative staffing profiles.

The Approach



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Optimization to determine set of shifts

Minimize $\sum \lambda_i [w_i(\lambda, \mu, p)] \sum p_i$

Subject to,

$A x = p$

$c x \leq H$

$x_j \geq 0$, integer for j=1,…J

$p_i \geq 0$, integer for i=1,…,24

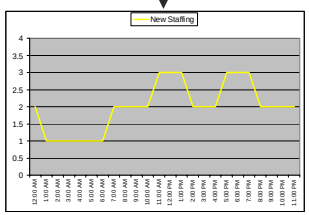
$p \geq p’$

Prescriptive Model - Advantages

- Linear integer program with a good structure.
- Extremely large problems can use column generation approaches.
- It’s a discrete optimization problem that most people can relate to.

The Approach

Simulation to help determine staffing profile for 24-hours



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Optimization to determine set of shifts

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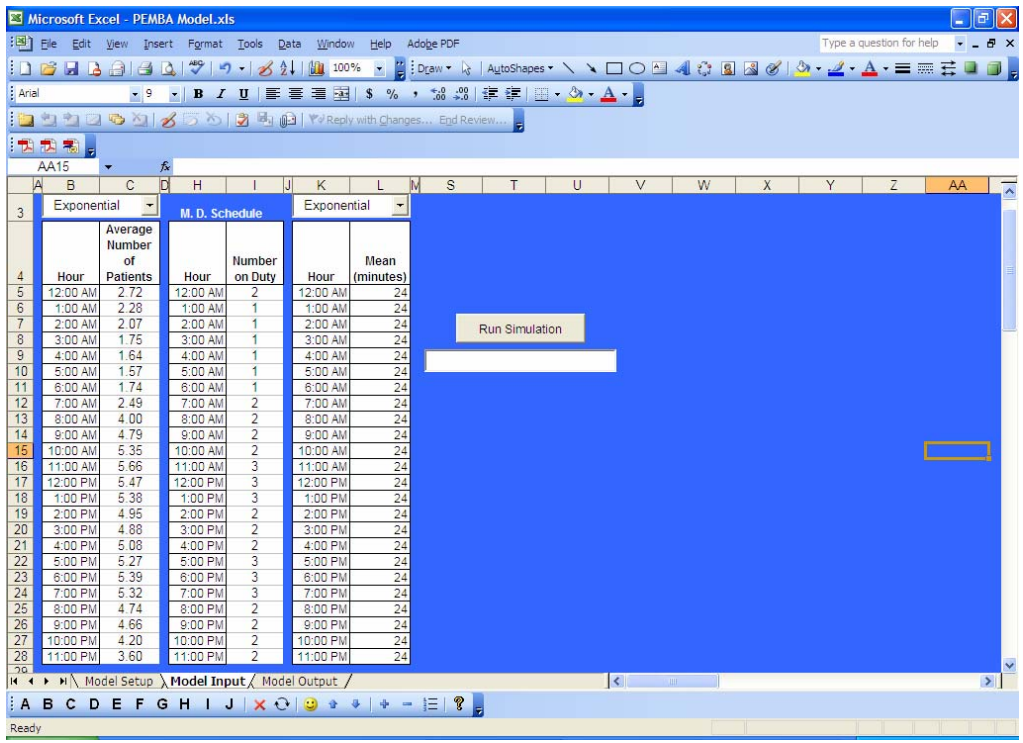
Implementation

Simulation to help determine staffing profile for 24-hours



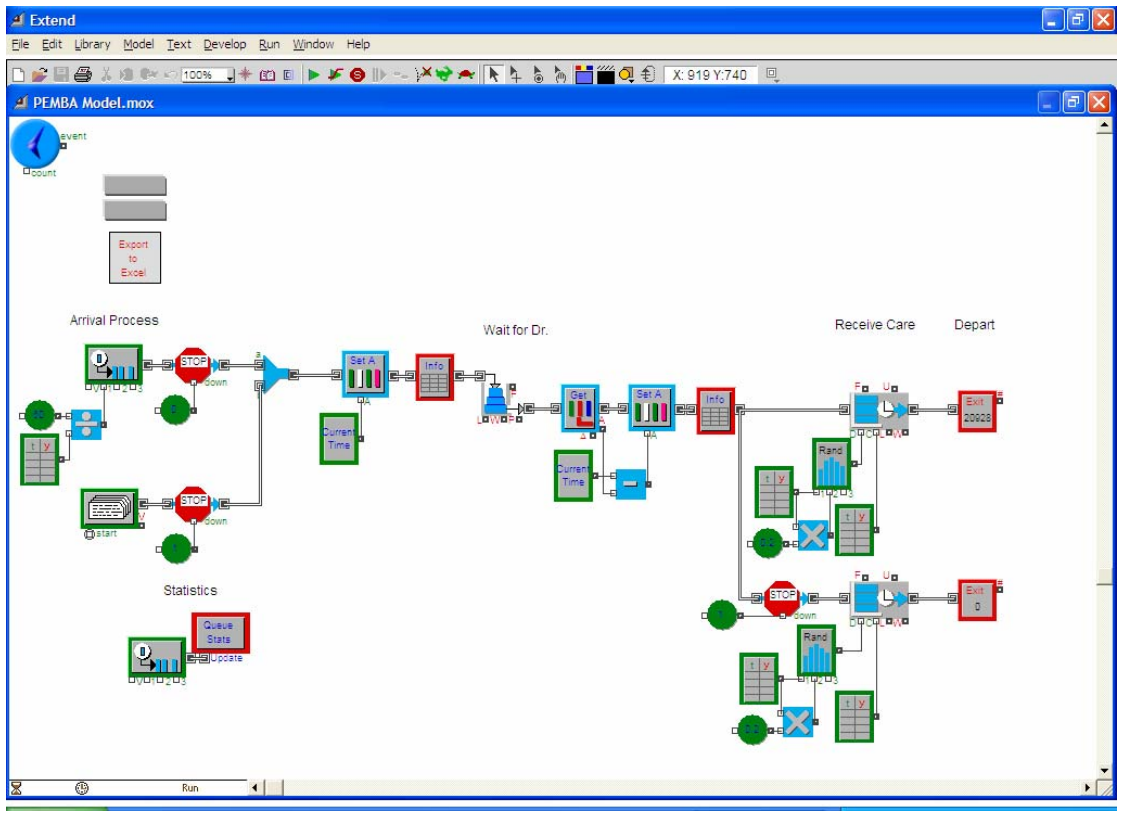
Hour	Average Number of Patients	Hour	Number on Duty	Hour	Mean (minutes)
12:00 AM	2.72	12:00 AM	2	12:00 AM	24
1:00 AM	2.28	1:00 AM	1	1:00 AM	24
2:00 AM	2.07	2:00 AM	1	2:00 AM	24
3:00 AM	1.75	3:00 AM	1	3:00 AM	24
4:00 AM	1.64	4:00 AM	1	4:00 AM	24
5:00 AM	1.57	5:00 AM	1	5:00 AM	24
6:00 AM	1.74	6:00 AM	1	6:00 AM	24
7:00 AM	2.49	7:00 AM	2	7:00 AM	24
8:00 AM	4.00	8:00 AM	2	8:00 AM	24
9:00 AM	4.79	9:00 AM	2	9:00 AM	24
10:00 AM	5.35	10:00 AM	2	10:00 AM	24
11:00 AM	5.66	11:00 AM	3	11:00 AM	24
12:00 PM	5.47	12:00 PM	3	12:00 PM	24
1:00 PM	5.38	1:00 PM	3	1:00 PM	24
2:00 PM	4.95	2:00 PM	2	2:00 PM	24
3:00 PM	4.88	3:00 PM	2	3:00 PM	24
4:00 PM	5.08	4:00 PM	2	4:00 PM	24
5:00 PM	5.27	5:00 PM	3	5:00 PM	24
6:00 PM	5.39	6:00 PM	3	6:00 PM	24
7:00 PM	5.32	7:00 PM	3	7:00 PM	24
8:00 PM	4.74	8:00 PM	2	8:00 PM	24
9:00 PM	4.66	9:00 PM	2	9:00 PM	24
10:00 PM	4.20	10:00 PM	2	10:00 PM	24
11:00 PM	3.60	11:00 PM	2	11:00 PM	24

Can also accommodate scheduled arrivals, other distributions, and two types of servers.

Implementation

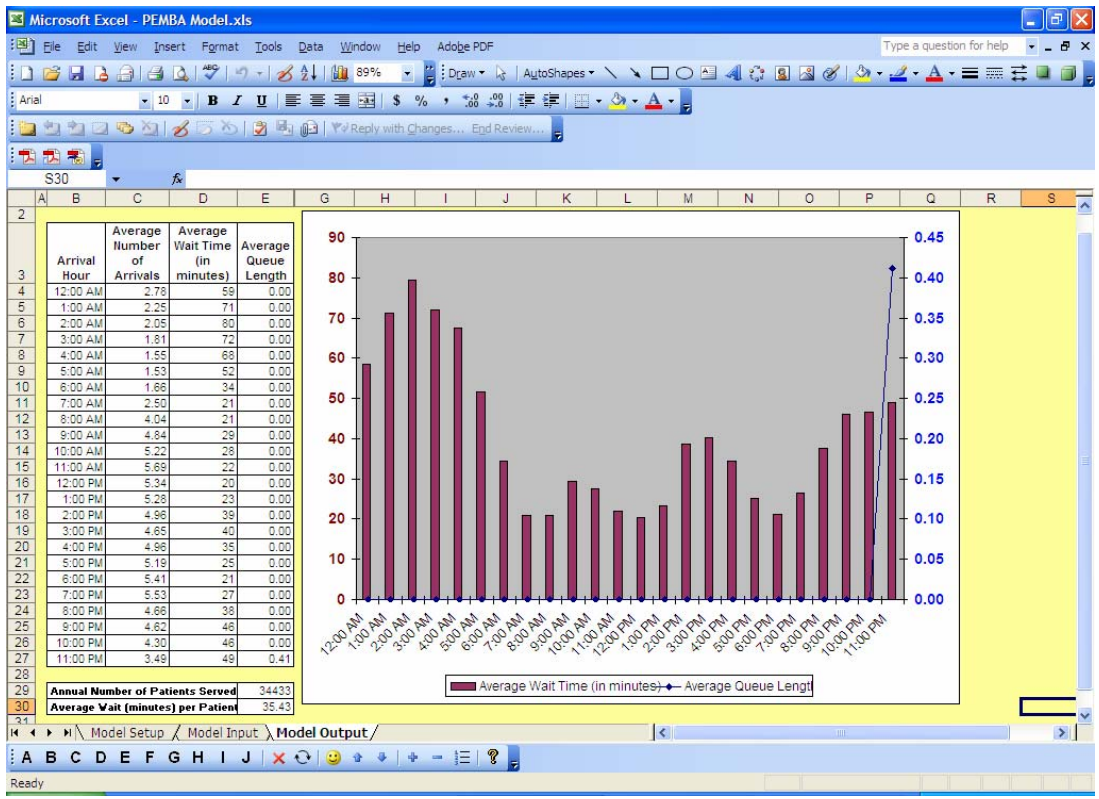
Simulation to help determine staffing profile for 24-hours

Excel utilizes the (free) Extend Simulation object.



Implementation

Simulation to help determine staffing profile for 24-hours

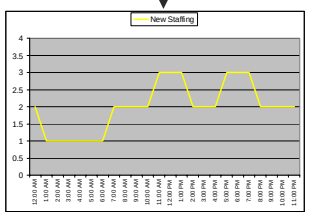


Arrival Hour	Average Number of Arrivals	Average Wait Time (in minutes)	Average Queue Length
12:00 AM	2.78	59	0.00
1:00 AM	2.25	71	0.00
2:00 AM	2.05	80	0.00
3:00 AM	1.81	72	0.00
4:00 AM	1.55	68	0.00
5:00 AM	1.53	52	0.00
6:00 AM	1.66	34	0.00
7:00 AM	2.50	21	0.00
8:00 AM	4.04	21	0.00
9:00 AM	4.84	29	0.00
10:00 AM	5.22	28	0.00
11:00 AM	5.69	22	0.00
12:00 PM	5.34	20	0.00
1:00 PM	5.28	23	0.00
2:00 PM	4.96	39	0.00
3:00 PM	4.65	40	0.00
4:00 PM	4.96	35	0.00
5:00 PM	5.19	25	0.00
6:00 PM	5.41	21	0.00
7:00 PM	5.53	27	0.00
8:00 PM	4.66	38	0.00
9:00 PM	4.62	46	0.00
10:00 PM	4.30	46	0.00
11:00 PM	3.49	49	0.41

Annual Number of Patients Served	34433
Average Wait (minutes) per Patient	35.43

Implementation

Simulation to help determine staffing profile for 24-hours



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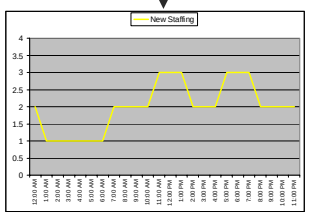
Optimization to determine set of shifts

Uses Excel’s Solver (or Premium Solver). Can easily incorporate additional constraints.

Implementation

Simulation to help determine staffing profile for 24-hours



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Optimization to determine set of shifts

Microsoft Excel - ShiftOptimization_3.xls

File Edit View Insert Format Tools Data Window Help Adobe PDF

C3 Needed Staff By Hour

Needed Staff and Desired Number of Shifts

Hour	Needed Staff By Hour
12:00 AM	2
1:00 AM	1
2:00 AM	1
3:00 AM	1
4:00 AM	1
5:00 AM	1
6:00 AM	1
7:00 AM	2
8:00 AM	2
9:00 AM	2
10:00 AM	2
11:00 AM	3
12:00 PM	3
1:00 PM	3
2:00 PM	2
3:00 PM	2
4:00 PM	2
5:00 PM	3
6:00 PM	3
7:00 PM	3
8:00 PM	2
9:00 PM	2
10:00 PM	2
11:00 PM	2

Minimum Desired Shifts	6
Maximum Desired Shifts	6

Continue

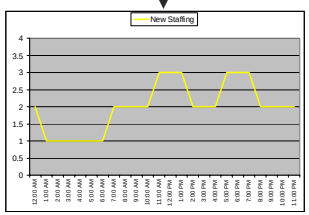
Clear

Close

Ready

Implementation

Simulation to help determine staffing profile for 24-hours



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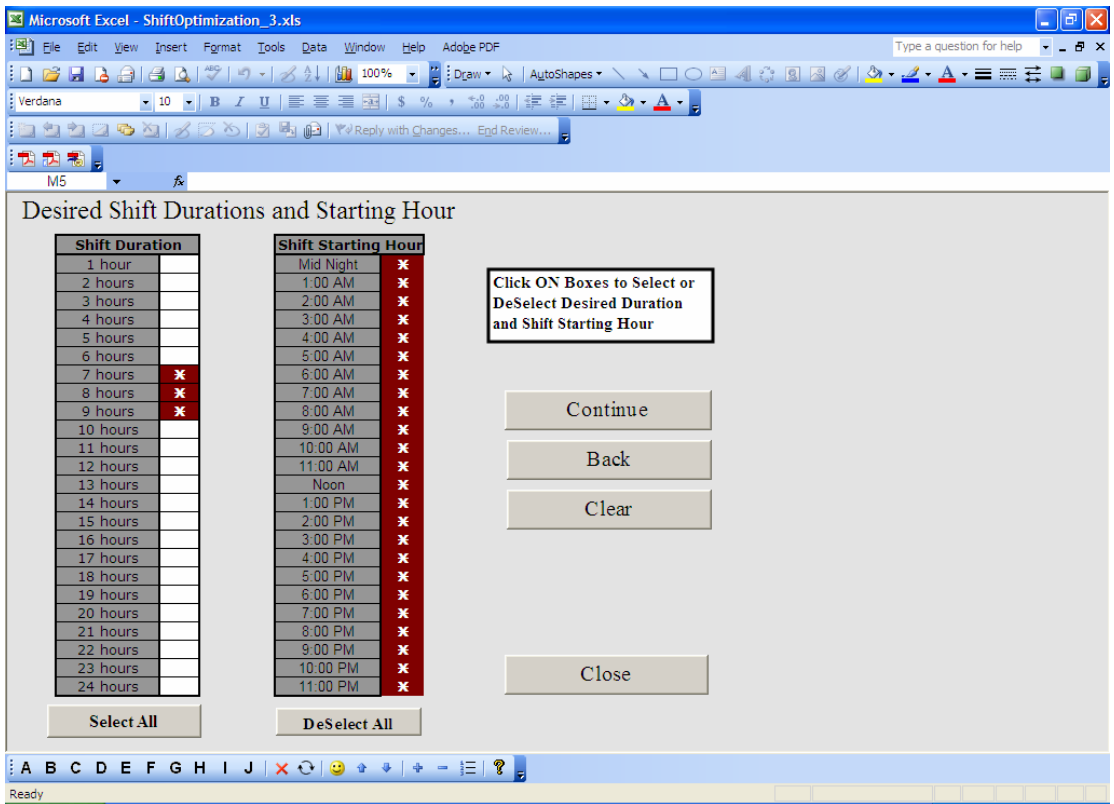

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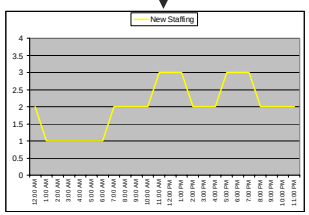

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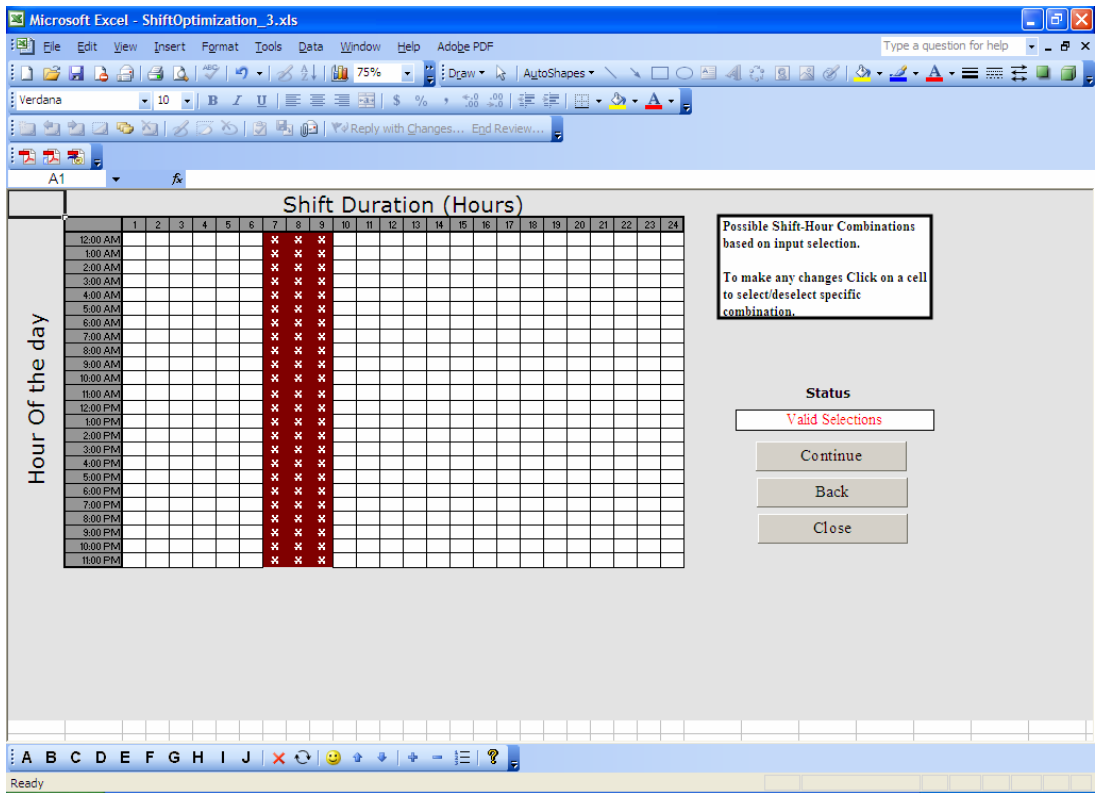

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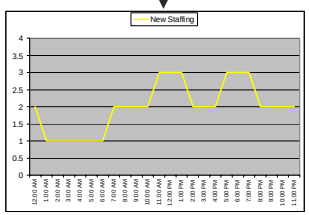

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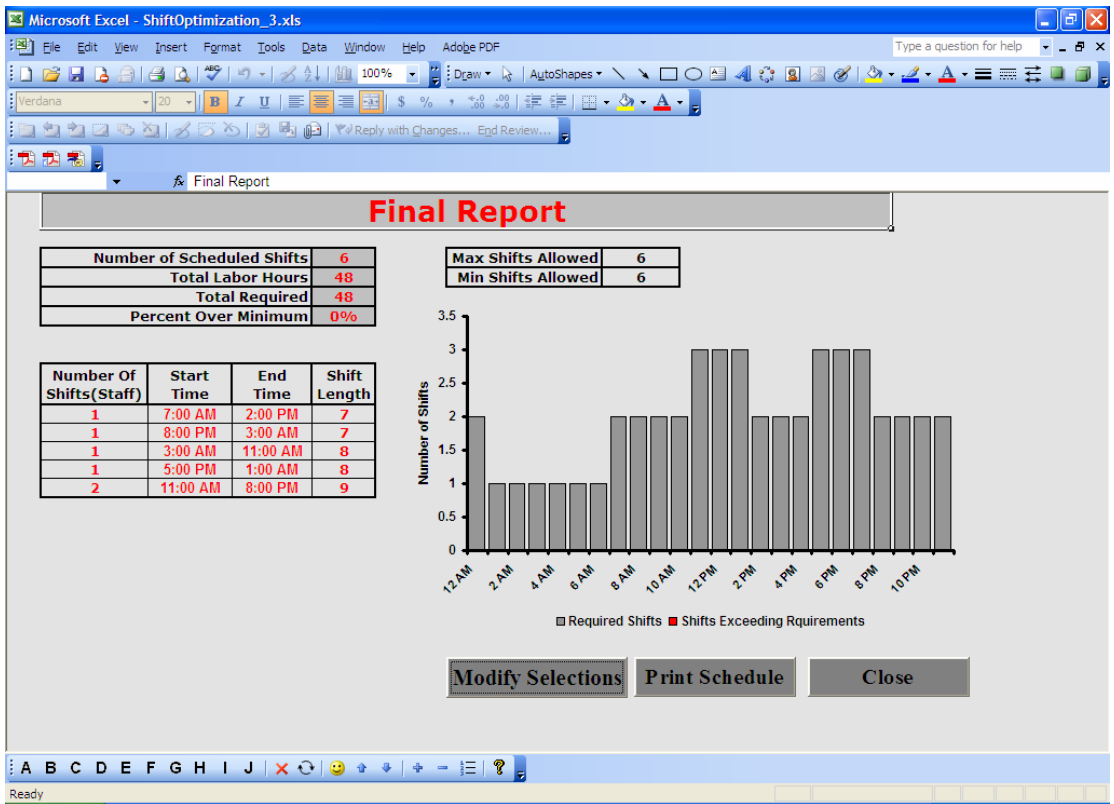
Optimization to determine set of shifts



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Results & Observations

- Example model uses
  - Los Angeles ER with 70 provider hours per day. Able to identify change of 1 shift to decrease estimated waiting time for provider by 26%.
  - Children's hospital ER in Montreal with 74 provider hours per day. Change several shifts able to decrease estimated waiting time for provider by 27%
  - Virginia ER with 118 provider hours per day. Strategically adds 6 hours and revises shift schedule to reduce estimated waiting time for provider by 30%.

Results & Observations

- The simulation helps bust the myth that patient waiting is a zero-sum game.
- Users seem to enjoy using the models.
- The models can help a decision-maker understand the tradeoffs between server schedules and patient waiting.
- Gains have been larger than originally expected.