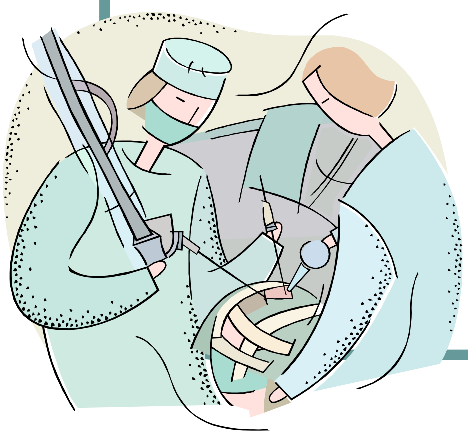
Planning and Scheduling of endoscopic activities by taking into account the availability of recovery beds

H.Fei, N.Meskens
Catholic University of Mons (FUCaM)

D.Duvivier
Université du Littoral Côte d'Opale

C.Combes
University of Jean Monnet (EURISE)



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Outline

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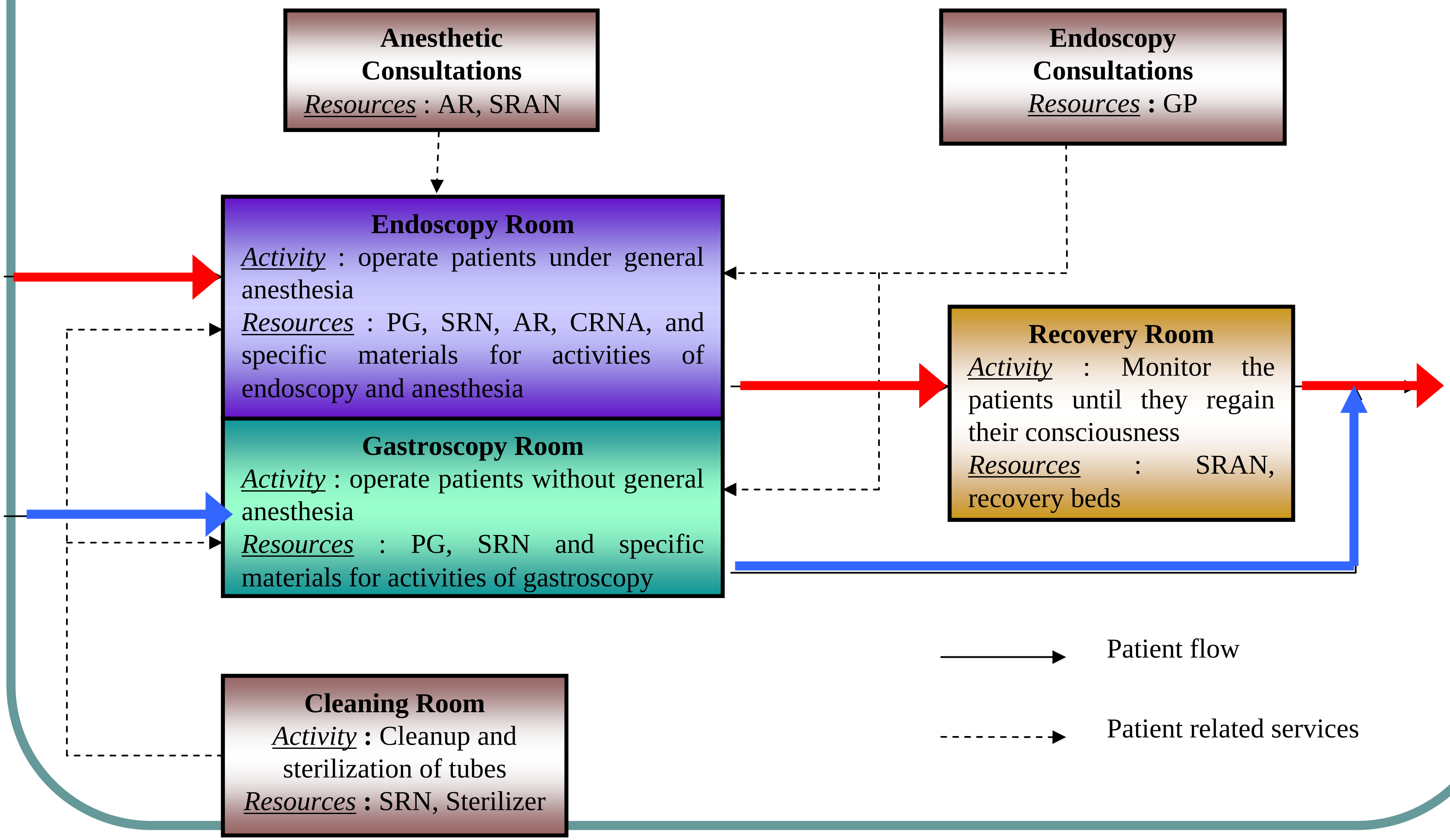
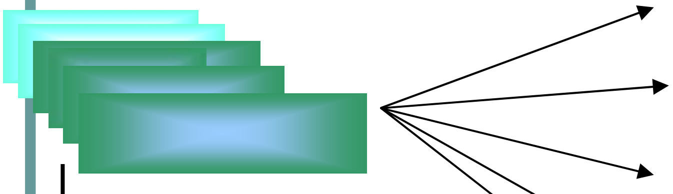
Background of this research

- Endoscopy center in « croix-Rousse » hospital of Lyon

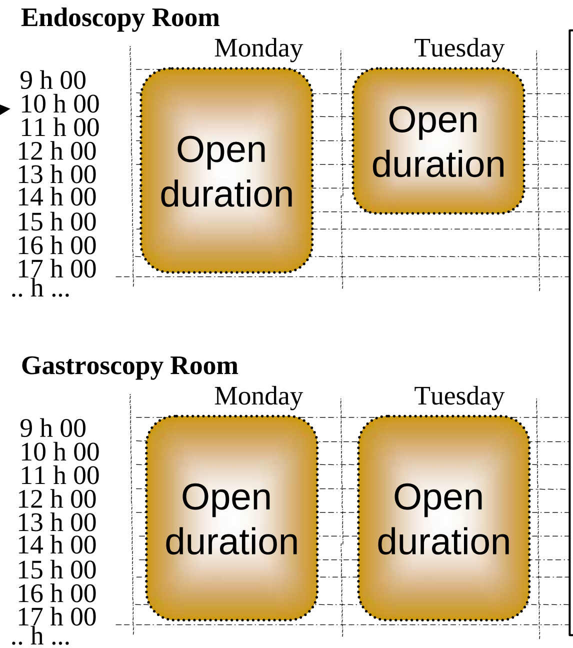


General description – General description of problem

Surgical cases



- *Maximal open duration of each operating room on one day;*
- *Allowed number of activities in the charge of one surgeon on one day;*
- *Deadline of Surgical case;*
- *Limited number of beds in the recovery room.*



OBJ:
to Construct a feasible and efficient operating program within one week with an objective of maximizing the operating room utilization and minimizing the total overtime operating cost

Hypotheses:

- Each patient, waiting for the operation, is assigned in advance to a specialized operating room and to a surgeon;
- Human and equipment resources, except surgeons and operating rooms, will be available whenever they are needed;
- Emergency cases are not taken into account and there is no pre-emption either;
- No surgeon can reserve one block of time in advance;
- All patients and surgeons are ready for their surgical cases.

General description - Framework of methodology

Problem proposed

A set of surgical cases waiting to be scheduled to the endoscopy center

Assign a set of surgical cases with respect of their deadline to each operating room for each day of the planning duration

Tactical endoscopy planning problem for one week

Determine the final operating order of the set of surgical cases assigned at the previous stage to a day and an operating room

Daily endsocopy scheduling problem

Final solution

Feasible opeating program for this endoscopy center for one week

Weekly planning - Mathematical model

$$\min \quad \sum_{j\in\Xi} C_j x_j$$

s.t.

$$\sum_{j\in\Xi} a_{ij} \left(\sum_{d=1}^{D_i}\sum_{k=1}^{N_O} b_j^d e_{kj}\right) x_j = 1,\ i\in\overline{\Omega}$$

$$\sum_{j\in\Xi} a_{ij} x_j \le 1,\ i\in \Omega \setminus \overline{\Omega}$$

$$\sum_{j\in\Xi} b_j^d e_{kj} x_j \le 1, \quad d\in\{1,\cdots,N_d\},\ k\in\{1,\cdots,N_O\}$$

$$\sum_{d=1}^{N_d} b_j^d x_j \le 1,\ j\in\Xi_k,\ k\in\{1,\cdots,N_O\}$$

$$\sum_{j\in\Xi} b_j^d \left(\sum_{i\in\Omega_l} a_{ij} g_i\right) x_j \le G_l,\ l\in\{1,\cdots,N_s\},\ d\in\{1,\cdots,N_d\}$$

$$\sum_{j\in\Xi} b_j^d \left(\sum_{i\in\Omega_l} a_{ij} t_i\right) x_j \le \max\{R_k^d + S_k^d,\ k=1,\cdots,N_O\}, \quad l=1,\cdots,N_s,\ \ d=1,\cdots,N_d$$

$$\sum_{d=1}^{N_d}\sum_{k=1}^{N_O} b_j^d e_{kj} \left(\sum_{i\in\Omega} a_{ij} t_i\right) x_j \le R_k^d + S_k^d \quad j\in\Xi,\ d\in\{1,\cdots,N_d\},\ k\in\{1,\cdots,N_O\}$$

$$x_j \in\{0,1\},\ j\in\Xi$$

where

$$C_j = \max\left\{ \left(\sum_{d=1}^{N_d}\sum_{k=1}^{N_O} b_j^d e_{kj} R_k^d - \sum_{i\in\Omega} a_{ij} t_i\right), \beta\left(\sum_{i\in\Omega} a_{ij} t_i - \sum_{d=1}^{N_d}\sum_{k=1}^{N_R} b_j^d e_{kj} R_k^d\right)\right\},\ j\in\Xi$$

$$\min \sum_{j\in\Xi} C_j x_j$$

s.t.

$$\sum_{j\in\Xi} a_{ij} \left(\sum_{d=1}^{D_i}\sum_{k=1}^{N_O} b_j^d e_{kj}\right) x_j = 1,\ i \in \overline{\Omega}$$

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where

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Minimization of the total cost of unexploited time and overtime

Weekly planning - Mathematical model

$$\min \quad \sum_{j\in\Xi} C_j x_j$$

s.t.

$$\sum_{j\in\Xi} a_{ij}\left(\sum_{d=1}^{D_i}\sum_{k=1}^{N_O} b_j^d e_{kj}\right)x_j = 1, i\in\overline{\Omega}$$

$$\sum_{j\in\Xi} a_{ij} x_j \le 1, i\in\Omega\setminus\overline{\Omega}$$

$$\sum_{j\in\Xi} b_j^d e_{kj} x_j \le 1, \quad d\in\{1,\cdots,N_d\}, k\in\{1,\cdots,N_O\}$$

$$\sum_{d=1}^{N_d} b_j^d x_j \le 1, \; j\in\Xi_k, k\in\{1,\cdots,N_O\}$$

$$\sum_{j\in\Xi} b_j^d\left(\sum_{i\in\Omega_l} a_{ij} g_i\right)x_j \le G_l, l\in\{1,\cdots,N_s\}, d\in\{1,\cdots,N_d\}$$

$$\sum_{j\in\Xi} b_j^d\left(\sum_{i\in\Omega_l} a_{ij} t_i\right)x_j \le \max\{R_k^d + S_k^d, k=1,\cdots,N_O\}, \quad l=1,\cdots,N_s, \quad d=1,\cdots,N_d$$

$$\sum_{d=1}^{N_d}\sum_{k=1}^{N_O} b_j^d e_{kj}\left(\sum_{i\in\Omega} a_{ij} t_i\right)x_j \le R_k^d + S_k^d \quad j\in\Xi, d\in\{1,\cdots,N_d\}, k\in\{1,\cdots,N_O\}$$

$$x_j \in\{0,1\}, j\in\Xi$$

where

$$C_j = \max\left\{\left(\sum_{d=1}^{N_d}\sum_{k=1}^{N_O} b_j^d e_{kj} R_k^d - \sum_{i\in\Omega} a_{ij} t_i\right), \beta\left(\sum_{i\in\Omega} a_{ij} t_i - \sum_{d=1}^{N_d}\sum_{k=1}^{N_R} b_j^d e_{kj} R_k^d\right)\right\}, j\in\Xi$$

Each surgical case should be treated exactly or at most once before the end of this week according to their deadlines

Weekly planning - Mathematical model

$$\min \quad \sum_{j\in\Xi} C_j x_j$$

s.t.

$$\sum_{j\in\Xi} a_{ij}\left(\sum_{d=1}^{D_i}\sum_{k=1}^{N_O} b_j^d e_{kj}\right)x_j = 1,\ i\in\overline{\Omega}$$

$$\sum_{j\in\Xi} a_{ij} x_j \le 1,\ i \in \Omega\setminus\overline{\Omega}$$

$$\sum_{j\in\Xi} b_j^d e_{kj} x_j \le 1, \quad d\in\{1,\cdots,N_d\},\ k\in\{1,\cdots,N_O\}$$

$$\sum_{d=1}^{N_d} b_j^d x_j \le 1,\ j\in\Xi_k, k\in\{1,\cdots,N_O\}$$

$$\sum_{j\in\Xi} b_j^d\left(\sum_{i\in\Omega_l} a_{ij} g_i\right)x_j \le G_l,\ l\in\{1,\cdots,N_s\},\ d\in\{1,\cdots,N_d\}$$

$$\sum_{j\in\Xi} b_j^d\left(\sum_{i\in\Omega_l} a_{ij} t_i\right)x_j \le \max\{R_k^d + S_k^d, k=1,\cdots,N_O\}, \quad l$$

$$\sum_{d=1}^{N_d}\sum_{k=1}^{N_O} b_j^d e_{kj}\left(\sum_{i\in\Omega} a_{ij} t_i\right)x_j \le R_k^d + S_k^d \quad j\in\Xi, d\in\{1,\cdots,N_d\}, k\in\{1,\cdots,N_O\}$$

$$x_j \in\{0,1\},\ j\in\Xi$$

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Relation between an opearting room and a constructed feasible plan

Weekly planning - Mathematical model

$$\min \quad \sum_{j\in\Xi} C_j x_j$$

s.t.

$$\sum_{j\in\Xi} a_{ij} \left(\sum_{d=1}^{D_i}\sum_{k=1}^{N_O} b_j^d e_{kj}\right) x_j = 1, i \in \overline{\Omega}$$

$$\sum_{j\in\Xi} a_{ij} x_j \le 1, i \in \Omega \setminus \overline{\Omega}$$

$$\sum_{j\in\Xi} b_j^d e_{kj} x_j \le 1, \quad d \in \{1,\cdots,N_d\}, k \in \{1,\cdots,N_O\}$$

$$\sum_{d=1}^{N_d} b_j^d x_j \le 1, j \in \Xi_k, k \in \{1,\cdots,N_O\}$$

$$\sum_{j\in\Xi} b_j^d \left(\sum_{i\in\Omega_l} a_{ij} g_i\right) x_j \le G_l, l \in \{1,\cdots,N_s\}, d \in \{1,\cdots,N_d\}$$

Capacity limitation of a surgeon on one day

$$\sum_{j\in\Xi} b_j^d \left(\sum_{i\in\Omega_l} a_{ij} t_i\right) x_j \le \max\{R_k^d + S_k^d, k = 1,\cdots,N_O\}, \quad l = 1,\cdots,N_s, \quad d = 1,\cdots,N_d$$

$$\sum_{d=1}^{N_d}\sum_{k=1}^{N_O} b_j^d e_{kj} \left(\sum_{i\in\Omega} a_{ij} t_i\right) x_j \le R_k^d + S_k^d \quad j \in \Xi, d \in \{1,\cdots,N_d\}, k \in \{1,\cdots,N_O\}$$

$$x_j \in \{0,1\}, j \in \Xi$$

where

$$C_j = \max\left\{\left(\sum_{d=1}^{N_d}\sum_{k=1}^{N_O} b_j^d e_{kj} R_k^d - \sum_{i\in\Omega} a_{ij} t_i\right), \beta\left(\sum_{i\in\Omega} a_{ij} t_i - \sum_{d=1}^{N_d}\sum_{k=1}^{N_R} b_j^d e_{kj} R_k^d\right)\right\}, j \in \Xi$$

Weekly planning - Mathematical model

$$\min \quad \sum_{j\in\Xi} C_j x_j$$

s.t.

$$\sum_{j\in\Xi} a_{ij}\left(\sum_{d=1}^{D_i}\sum_{k=1}^{N_O} b_j^d e_{kj}\right)x_j = 1,\ i\in\overline{\Omega}$$

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$$\sum_{d=1}^{N_d} b_j^d x_j \le 1,\ j\in\Xi_k,\ k\in\{1,\cdots,N_O\}$$

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$$\sum_{j\in\Xi} b_j^d\left(\sum_{i\in\Omega_l} a_{ij} t_i\right)x_j \le \max\{R_k^d + S_k^d,\ k=1,\cdots,N_O\},\quad l=1,\cdots,N_s,\quad d=1,\cdots,N_d$$

$$\sum_{d=1}^{N_d}\sum_{k=1}^{N_O} b_j^d e_{kj}\left(\sum_{i\in\Omega} a_{ij} t_i\right)x_j \le R_k^d + S_k^d \qquad j\in\Xi,\ d\in\{1,\cdots,N_d\},\ k\in\{1,\cdots,N_O\}$$

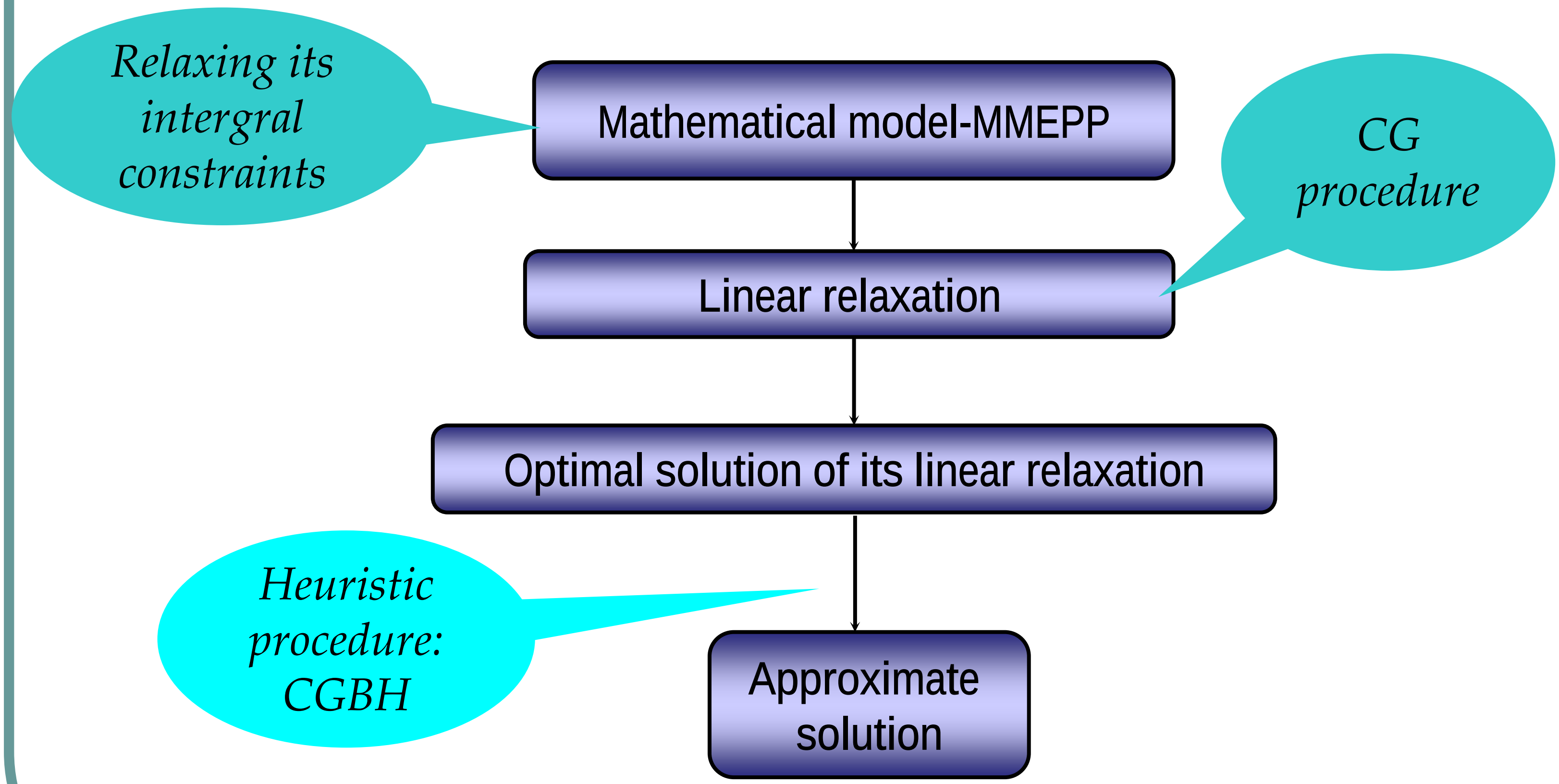
$$x_j\in\{0,1\},\ j\in\Xi$$

where

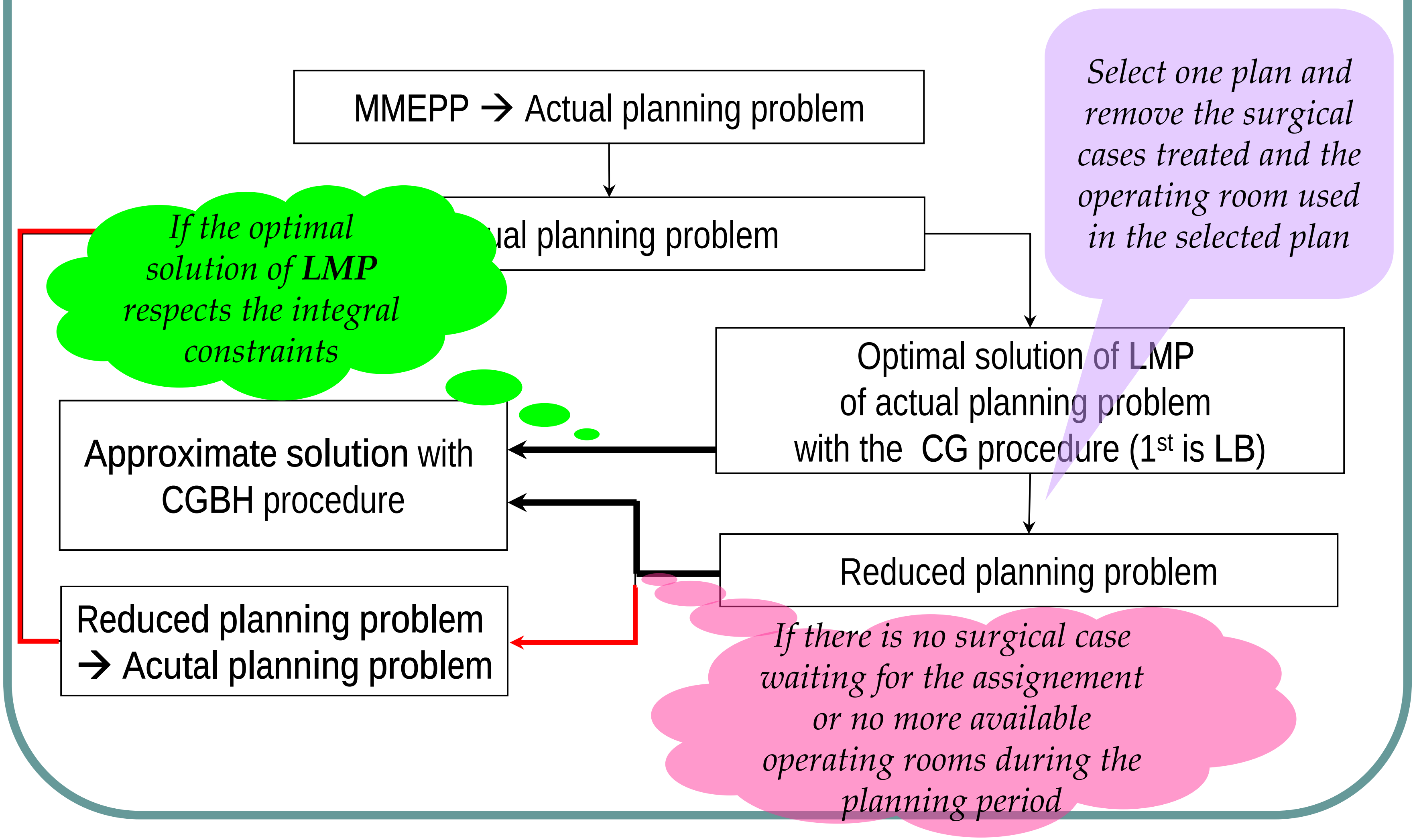
$$C_j = \max\left\{\left(\sum_{d=1}^{N_d}\sum_{k=1}^{N_O} b_j^d e_{kj} R_k^d - \sum_{i\in\Omega} a_{ij} t_i\right),\ \beta\left(\sum_{i\in\Omega} a_{ij} t_i - \sum_{d=1}^{N_d}\sum_{k=1}^{N_R} b_j^d e\right.\right.$$

Capacity limitation of an operating room

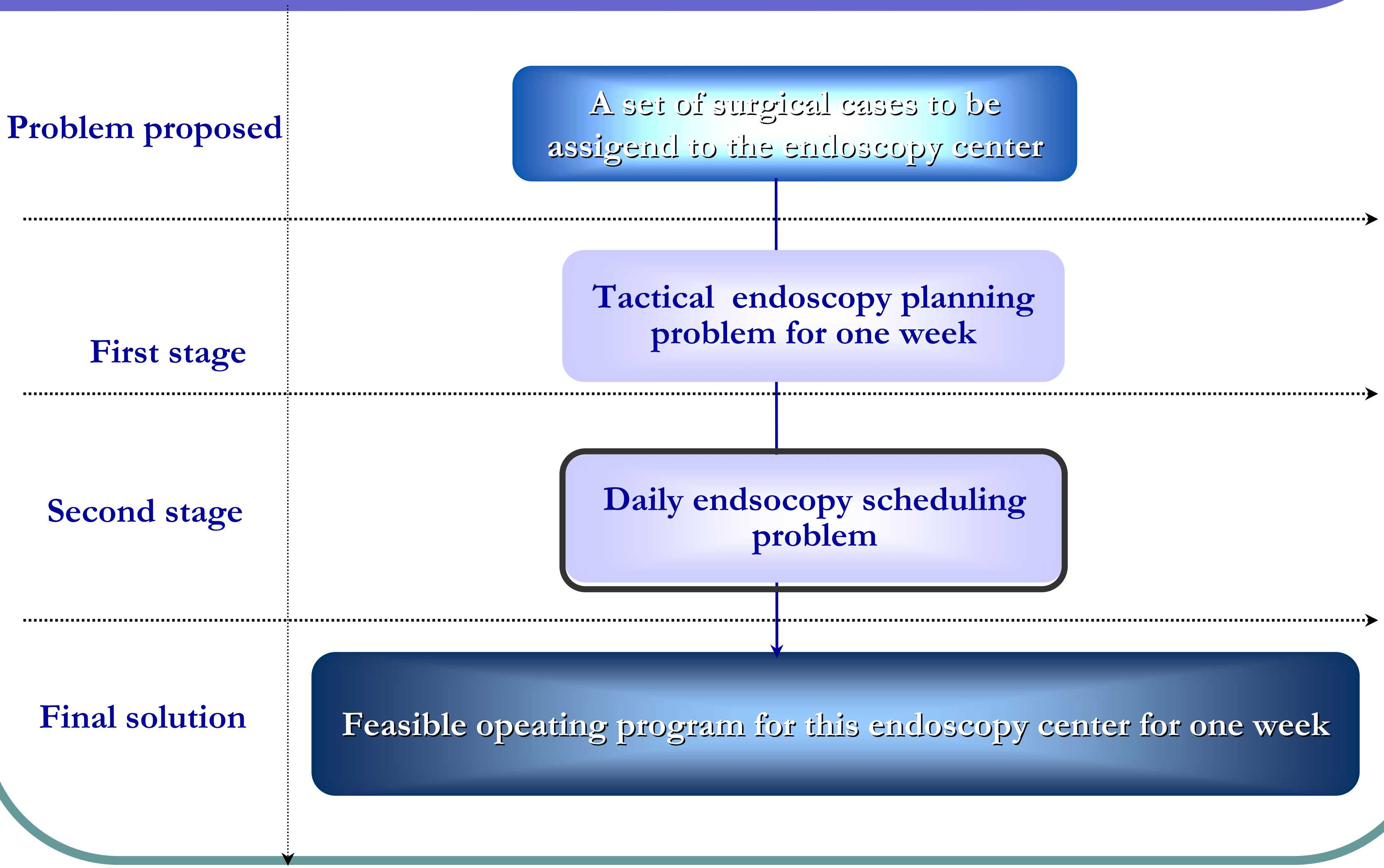
Weekly planning - Methodology



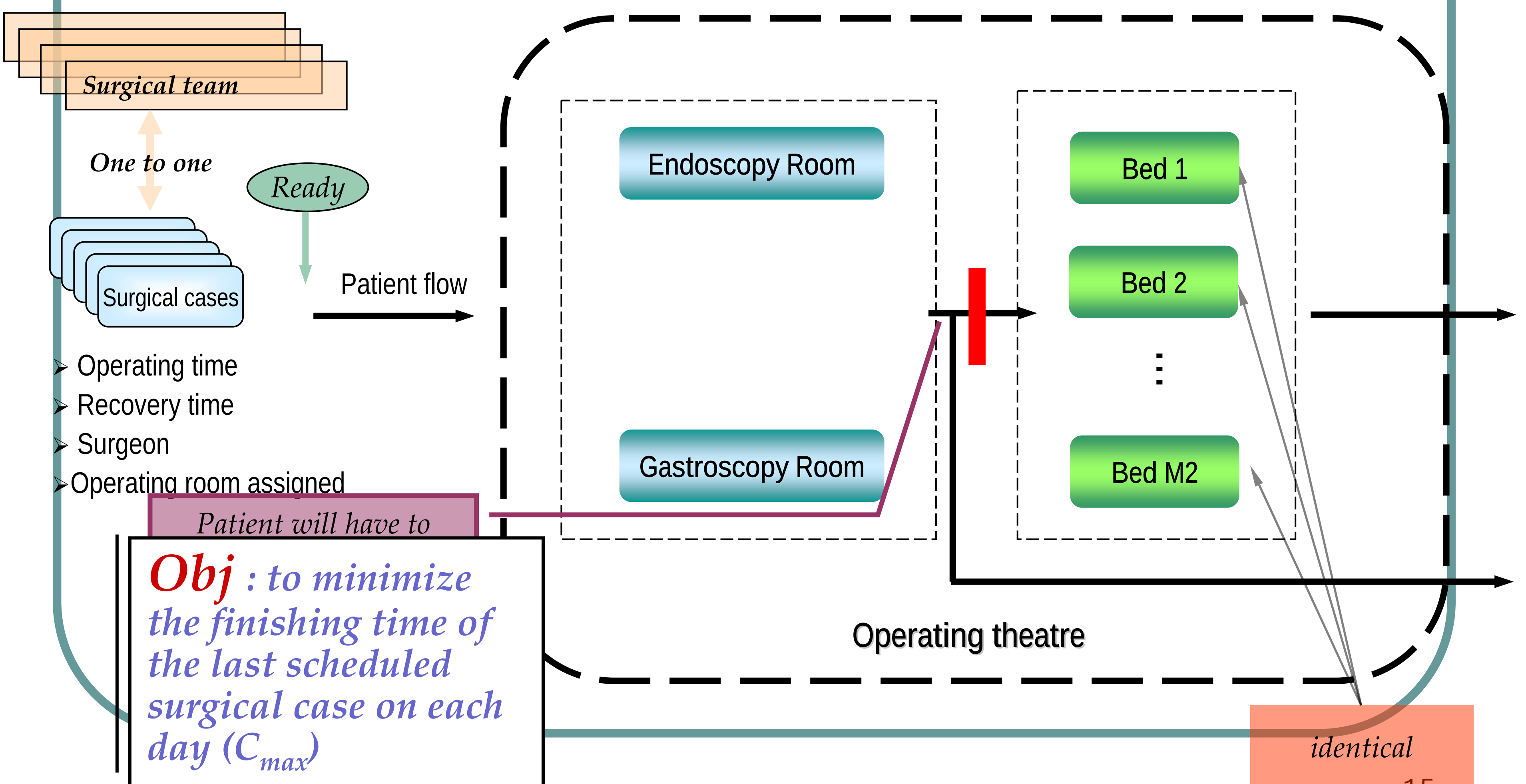
Weekly planning – CGBH procedure



Daily sheduling

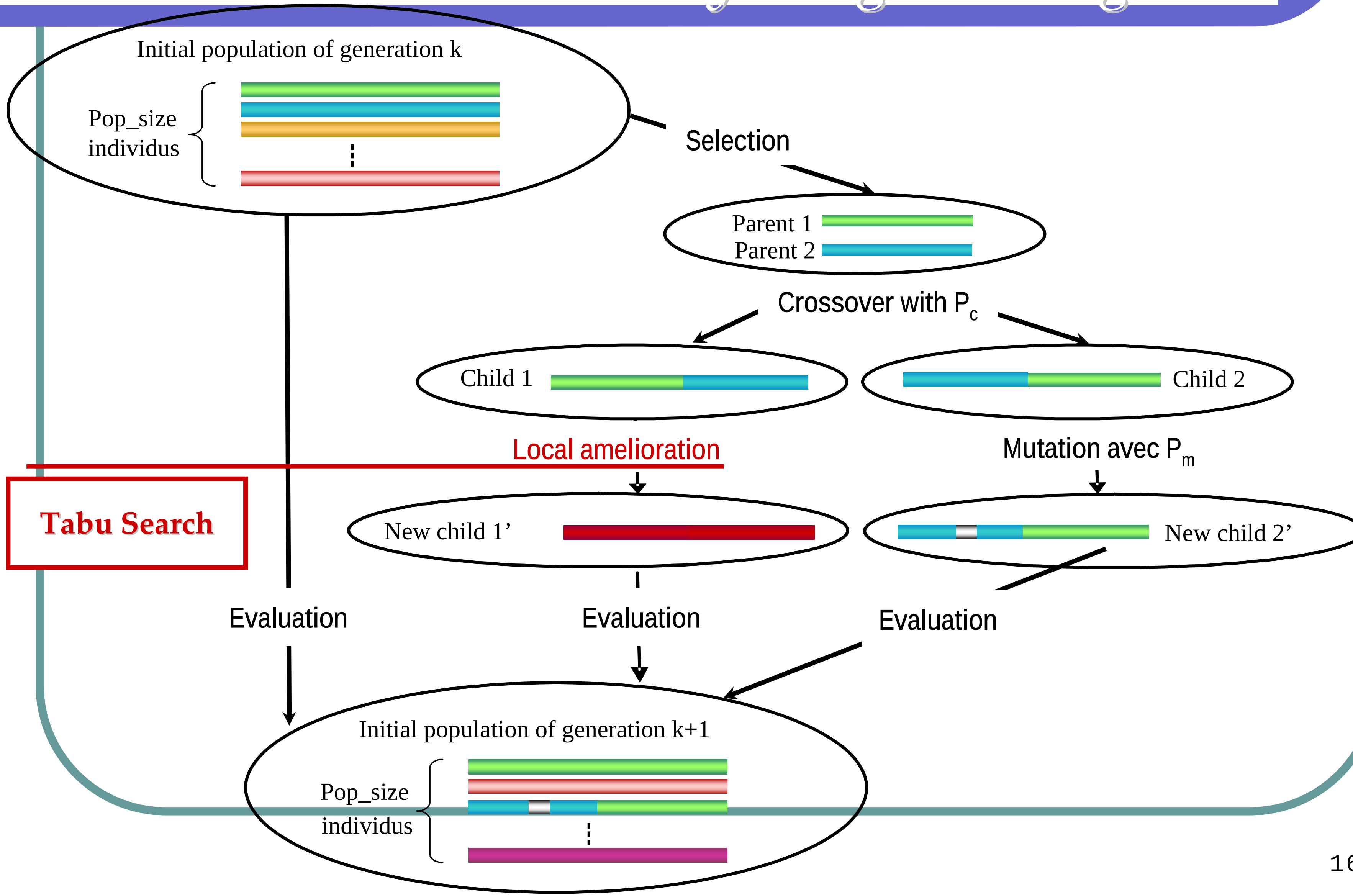


Daily sheduling – General description of scheduling problem



***Obj** : to minimize the finishing time of the last scheduled surgical case on each day (C_{max})*

Daily sheduling – Hybrid genetic algorithm



Experimental results – Requirement & data

Requirements

- Hardware: IBM ThinkPad T42 (CPU: PM, 1.6GHz, Memory:256Mb)
- Software : Mircosoft VC2005; COIN, a linear programming solver (http://www.coin-or.org/download.html)

Data

<table>
<tr><td>N_d</td><td colspan="3">5 (unity: day)</td><td colspan="2">Operating time (unity: minute)</td></tr>
<tr><td>N_s</td><td colspan="3">8; on each planning day, $NG_l = 11$</td><td rowspan="4">OR 1</td><td rowspan="4">The experimental data are randomly generated with the Pearson III distribution rule, where the parameters have been obtained from the real observation of activities of the endoscopy center concerned.</td></tr>
<tr><td>N</td><td colspan="3">80,90,100,110,120</td></tr>
<tr><td>D_i</td><td colspan="3">Uniform distribution: U[1,20]   (unity: day)</td></tr>
<tr><td>N_O</td><td>2 (an endoscopy room (OR1); a gastroscopy room (OR2))</td><td>N_b</td><td>3</td></tr>
<tr><td>R_k^d</td><td>240 (OR1), 420 (OR2) (unity: minute)</td><td rowspan="2">β</td><td rowspan="2">1.5</td><td rowspan="2">OR 2</td><td rowspan="2">Operating time of the same type of activities in the endsocopy room + U[5,10]</td></tr>
<tr><td>S_k^d</td><td>120 (unity: minute)</td></tr>
</table>

Experimental results

N(N1:N2)	ObjGS	ObjHGA	CPUGS	CPUHGA	dLBSch
80(30:50)	1680	1390.4	0	1.454	1001.45
80(40:40)	1399.6	1009.6	0	1.031	989.96
90(40:50)	2993.4	2854.4	0	8.823	2576.85
100(40:60)	3371.4	2865	0.01	9.956	2598.18
110(40:70)	3544.8	2965	0	10.546	2607.45
120(40:80)	3622	3060	0	9.734	2692.18
130(50:80)	3658.4	3194.4	0	13.038	2943.64

Conclusion: the hybrid genetic algorithm (HGA) operating program is more "dense" or has less unexpected idle time between surgical cases than the Gonzalez-Sahni (GS) operating program [Fei et al., 2006]!!

Conclusions & perspectives

Conclusions

- *A feasible operating program is presented for an endoscopy center obtained with the combination of the column-generation-based heuristic procedure and a hybrid genetic algorithm*;
- *The experimental results proves that the results obtained by the proposed hybrid genetic algorithm are generally better than those of the Gonzalez-Sahni algorithm that shows good performance in [fei et al., 2006] with regard to the operating cost.*

Perspectives

➢*The employed instances have not yet been validated in the hospital because more constraints (such as the limited capacity of PACU and so on) should be taken into account in the real case. The study thus will continue in this direction.*

➢*Unlike the Gonzalez-Sahni algorithm, the hybrid genetic algorithm proposed in this paper can also be employed to more than two operating rooms, so further study will be done while the number of operating rooms increases.*

Thanks for your attention