A genetic algorithm approach to the nurse scheduling problem with fuzzy preferences

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Outline

- Nurse Scheduling problem definition
- SEMOPS method
- GA definition
- Experimental results
- Conclusions and future work
Nurse Scheduling Problem

$N$ nurses to be scheduled
$M$ days to be scheduled
$w$ shift types
$g$ nurse grades

Schedules are represented in a matrix $M \times N$
## Nurse Scheduling Problem

### Table 1. Shift Types

<table>
<thead>
<tr>
<th>Shift</th>
<th>Start and end time</th>
<th>Symbol</th>
<th>( w )</th>
</tr>
</thead>
<tbody>
<tr>
<td>day-shift</td>
<td>(8:00-16:00)</td>
<td>( d )</td>
<td>1</td>
</tr>
<tr>
<td>evening-shift</td>
<td>(16:00-24:00)</td>
<td>( e )</td>
<td>2</td>
</tr>
<tr>
<td>night-shift</td>
<td>(00:00-8:00)</td>
<td>( n )</td>
<td>3</td>
</tr>
<tr>
<td>day off</td>
<td>---------------------</td>
<td>( o )</td>
<td>0</td>
</tr>
</tbody>
</table>

### Decision variable:

\[
x_{ijw} = \begin{cases} 
1 & \text{if nurse } i \text{ works shift } w \text{ on day } j \\
0 & \text{otherwise}
\end{cases}
\]
Constraints definition

**Hard constraints:**

- Each nurse can work only one shift a day

\[
\sum_{w=1}^{3} x_{ijw} = 1; \quad i = 1, \ldots, N; \quad j = 1, \ldots, M
\]

- The number of nurses needed per day is fulfilled

\[
R_{jwg} \leq \sum_{i=1}^{N} x_{ijw}; \quad j = 1, \ldots, M; \quad g = 1, 2, 3; \quad w = 1, 2, 3
\]

- Each nurse must have two days off per week

\[
\sum_{j=1}^{M} x_{ijw} = 5; \quad i = 1, \ldots, N; \quad w = 1, 2, 3
\]
Constraints definition

**Soft constraints:**

1. After a night shift a nurse prefers not to have a day shift
2. The schedule has to be seen as fair
3. Maximum number of consecutive working days is 4
4. Minimum number of consecutive working days is 1
5. Maximum number of night shifts is 3
Objectives definition

Objective 1: maximisation of entire schedule fitness

Table 2. Assigned fitness values

<table>
<thead>
<tr>
<th>Days pattern</th>
<th>Assigned penalty value</th>
</tr>
</thead>
<tbody>
<tr>
<td>n d</td>
<td>$p_1$</td>
</tr>
<tr>
<td>n e</td>
<td>$p_2$</td>
</tr>
<tr>
<td>e d</td>
<td>$p_3$</td>
</tr>
</tbody>
</table>

The individual schedule total penalty $P_i$:

$$P_i = \sum_{t=1}^{M-1} p_{it}$$
Objectives definition

Membership function of fuzzy individual schedule fitness:

$$\mu_{I_i}(P_i) = \begin{cases} 
1 & \text{if } P_i \leq p_i^L \\
1 - \frac{P_i - p_i^L}{p_i^U - p_i^L} & \text{if } p_i^L < P_i < p_i^U \\
0 & \text{if } p_i^U \leq P_i 
\end{cases}$$
Objective 1: maximisation of entire schedule fitness

to maximise the minimum degree level of satisfaction with respect to individual schedule penalties

maximise \( T \),

where \( T = \min_{i=1,\ldots,N} (\mu_{I_i}(P_i)) \)
Objectives definition

**Objective 2:** minimisation of entire schedule penalty for breaking the number of consecutive working days soft constraints

\[ NC = \sum_{i=1}^{N} wc_i \]

\[ wc_i = \begin{cases} 
0 & \text{if nurse } i \text{ works } 1 \leq c_i \leq 4 \\
1 & \text{otherwise}
\end{cases} \]
**Objective 3**: minimisation of entire schedule penalty for having nurses working more than 3 night shifts

\[ NS = \sum_{i=1}^{N} w_{ni} \]

\[ w_{ni} = \begin{cases} 
1 & \text{if nurse } i \text{ works } r_i > 3 \\
0 & \text{otherwise}
\end{cases} \]
**SEMOPS method**

*Sequential Multiobjective Problem Solving*

\[ z = (z_1, z_2, \ldots, z_p) \]

**At most:**

\[ z_i(x) \leq AL_i; d_i = \frac{z_i(x)}{AL_i} \]

**At least:**

\[ z_i(x) \geq AL_i; d_i = \frac{AL_i}{z_i(x)} \]

**Surrogate objective function:**

\[ s = \sum_{p=1}^{P} d_p \]
Objectives definition

Membership function of fuzzy individual schedule fitness:

\[
\mu_{I_i}(P_i) = \begin{cases} 
1 & \text{if } P_i \leq p_i^L \\
1 - \frac{p_i - p_i^L}{p_i^U - p_i^L} & \text{if } p_i^L < P_i < p_i^U \\
-0.5 \frac{P_i - p_i^U}{p_i^U} & \text{if } p_i^U \leq P_i
\end{cases}
\]
Multi-objective genetic algorithm

- Decision Problem Objectives
- DM Aspiration levels
- Fitness Function
  - GA
  - Solutions
Genetic Algorithm definition

• Binary representation
• Binary tournament selection
• Two-point crossover
• Four-point crossover
• Population size 960
• $T$ number of generations
### Experimental results

1000 generations, two-point crossover, Pmutation=0.1

#### SCHEDULE

<table>
<thead>
<tr>
<th>Nurse</th>
<th>day 1</th>
<th>day 2</th>
<th>day 3</th>
<th>day 4</th>
<th>day 5</th>
<th>day 6</th>
<th>day 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>nurse 1</td>
<td>e</td>
<td>e</td>
<td>e</td>
<td>n</td>
<td>o</td>
<td>e</td>
<td>o</td>
</tr>
<tr>
<td>nurse 2</td>
<td>e</td>
<td>e</td>
<td>o</td>
<td>d</td>
<td>d</td>
<td>o</td>
<td>d</td>
</tr>
<tr>
<td>nurse 3</td>
<td>d</td>
<td>n</td>
<td>n</td>
<td>n</td>
<td>o</td>
<td>o</td>
<td>e</td>
</tr>
<tr>
<td>nurse 4</td>
<td>d</td>
<td>n</td>
<td>n</td>
<td>o</td>
<td>o</td>
<td>e</td>
<td>n</td>
</tr>
<tr>
<td>nurse 5</td>
<td>o</td>
<td>d</td>
<td>e</td>
<td>d</td>
<td>n</td>
<td>o</td>
<td>n</td>
</tr>
<tr>
<td>nurse 6</td>
<td>e</td>
<td>d</td>
<td>o</td>
<td>d</td>
<td>d</td>
<td>o</td>
<td></td>
</tr>
<tr>
<td>nurse 7</td>
<td>o</td>
<td>d</td>
<td>d</td>
<td>e</td>
<td>d</td>
<td>o</td>
<td>n</td>
</tr>
<tr>
<td>nurse 8</td>
<td>d</td>
<td>o</td>
<td>e</td>
<td>o</td>
<td>d</td>
<td>n</td>
<td>e</td>
</tr>
</tbody>
</table>

\[ d_1 = 1; d_2 = 0; d_3 = 0; \]
Experimental results

1000 generations, four-point crossover, \( P_{\text{mutation}} = 0.1 \)

<table>
<thead>
<tr>
<th>SCHEDULE</th>
<th>day 1</th>
<th>day 2</th>
<th>day 3</th>
<th>day 4</th>
<th>day 5</th>
<th>day 6</th>
<th>day 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>nurse 1</td>
<td>e</td>
<td>o</td>
<td>d</td>
<td>n</td>
<td>e</td>
<td>d</td>
<td>o</td>
</tr>
<tr>
<td>nurse 2</td>
<td>e</td>
<td>e</td>
<td>o</td>
<td>n</td>
<td>o</td>
<td>e</td>
<td>n</td>
</tr>
<tr>
<td>nurse 3</td>
<td>e</td>
<td>e</td>
<td>o</td>
<td>o</td>
<td>e</td>
<td>e</td>
<td>e</td>
</tr>
<tr>
<td>nurse 4</td>
<td>o</td>
<td>d</td>
<td>e</td>
<td>n</td>
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<td>nurse 6</td>
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<td>n</td>
<td>o</td>
<td>n</td>
</tr>
</tbody>
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\( d_1 = 0.875; d_2 = 0; d_3 = 0; \)
Conclusion and Future Work

• The approach proposed has two main characteristics:
  • The nurse scheduling problem is defined as a multi-objective problem with fuzzy individual schedule fitness
  • A hybrid approach based on an interactive sequential method combined with a genetic algorithms is developed

• The use of fuzzy sets is beneficial when subjective judgement is appropriate to use. Individual nurses preferences
Future Work

- The Head nurse’s preferences can be modelled using fuzzy sets
- Linguistically quantified statements can be also used to prioritise the objectives
- Other constraints can be added to the problem
Nurse Scheduling Problem

Table  Nurses required in a day

<table>
<thead>
<tr>
<th>Shift</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>day-shift</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>evening-shift</td>
<td>0</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>night-shift</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
The following set and vectors are defined:

Candidate solutions \( X = \{ x_1, x_2, \ldots, x_V \} \)

Aspirations levels \( AL = [AL_1, AL_2, \ldots, AL_N] \) where \( N \) is the number of objectives.

Vector of distances between the aspiration levels and objective values

\[ \Delta_{x_v} = [\Delta_{x_v,1}, \Delta_{x_v,2}, \ldots, \Delta_{x_v,N}] \]
Fitness function definition

Fuzzy set $A = \text{acceptable distance between the objective value and the aspiration level}$

The membership function $\mu_A$ represents the degree to which an achieved objective value satisfies the DM with respect to its distance from the aspiration level $AL_n$. 

![Graph showing the membership function $\mu_A(\Delta)$]
Fitness function definition

A linguistically quantified statement = “most distances between the achieved objective values and the aspiration levels are acceptable”

"\( Q A_{x_v}'s \) are \( A \)"

Fitness function \( D \) is defined as:

\[
D(x_v) = \text{Truth}["Q A_{x_v}'s are } A"] = \mu_Q(r_{x_v}) \in [0, 1]
\]
Yager’s algebraic approach:

\[ r_{x,v} = \frac{1}{N} \sum_{n=1}^{N} \mu_A (\Delta_{x,v}, n) \]

For the quantifier “most”:

\[ \mu_Q (r_{x,v}) = e^{-50(r_{x,v} - 1)^2} \]
Fitness function definition

Find $x_v^* \in X$ that maximises the degree of truth of the linguistically quantified fitness function:

$$D(x_v^*) = \max_{x_v \in X} D(x_v)$$