A genetic algorithm approach to the nurse scheduling problem with fuzzy preferences

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Outline

- Nurse Scheduling problem definition
- SEMOPS method
- GA definition
- Experimental results
- Conclusions and future work

Nurse Scheduling Problem

N nurses to be scheduled
M days to be scheduled
w shift types
g nurse grades

Schedules are represented in a matrix M x N

Nurse Scheduling Problem

Table 1. Shift Types

Shift	Start and end time	Symbol	w
day-shift	(8:00-16:00)	d	1
evening-shift	(16:00-24:00)	e	2
night-shift	(00:00-8:00)	n	3
day off	----------------	o	0

Decision variable:

$$x_{ijw} = \begin{cases} 1 & \text{if nurse } i \text{ works shift } w \text{ on day } j \\ 0 & \text{otherwise} \end{cases}$$

Constraints definition

Hard constraints:

- Each nurse can work only one shift a day

$$\sum_{w=1}^{3} x_{ijw} = 1; \; i = 1,\ldots, N; \; j = 1,\ldots, M$$

- The number of nurses needed per day is fulfilled

$$R_{jwg} \leq \sum_{i=1}^{N} x_{ijw} \; ; j = 1, \ldots, M; \; g = 1, 2, 3; \; w = 1, 2, 3$$

- Each nurse must have two days off per week

$$\sum_{j=1}^{M} x_{ijw} = 5; \; i = 1,\ldots, N; \; w = 1, 2, 3$$

Constraints definition

Soft constraints:

1. After a night shift a nurse prefers not to have a day shift
2. The schedule has to be seen as fair
3. Maximum number of consecutive working days is 4
4. Minimum number of consecutive working days is 1
5. Maximum number of night shifts is 3

Objectives definition

Objective 1: maximisation of entire schedule fitness

Table 2. Assigned fitness values

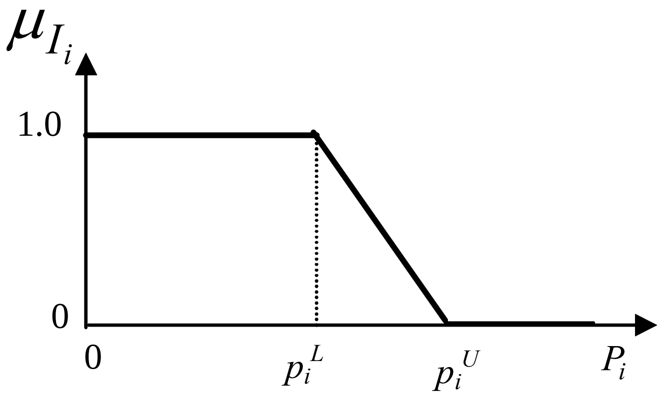
Days pattern	Assigned penalty value
n d	p_1
n e	p_2
e d	p_3

The individual schedule total penalty P_i :

$$P_i = \sum_{t=1}^{M-1} p_{it}$$

Objectives definition

Membership function of fuzzy individual schedule fitness:



$$\mu_{I_i}(P_i) = \begin{cases} 1 & \text{if} \quad P_i \le p_i^L \\ 1 - \dfrac{P_i - p_i^L}{p_i^U - p_i^L} & \text{if} \quad p_i^L < P_i < p_i^U \\ 0 & \text{if} \quad p_i^U \le P_i \end{cases}$$

Objectives definition

Objective 1: maximisation of entire schedule fitness

to maximise the minimum degree level of satisfaction with respect to individual schedule penalties

$$\text{maximise } T,$$
$$\text{where } T = \min_{i=1,\ldots,N} (\mu_{I_i}(P_i))$$

Objectives definition

Objective 2: minimisation of entire schedule penalty for breaking the number of consecutive working days soft constraints

$$NC = \sum_{i=1}^{N} wc_i$$

$$wc_i = \begin{cases} 0 & \text{if nurse } i \text{ works } 1 \leq c_i \leq 4 \\ 1 & \text{otherwise} \end{cases}$$

Objectives definition

Objective 3: minimisation of entire schedule penalty for having nurses working more than 3 night shifts

$$NS = \sum_{i=1}^{N} wn_i$$

$$wn_i = \begin{cases} 1 & \text{if nurse } i \text{ works } r_i > 3 \\ 0 & \text{otherwise} \end{cases}$$

SEMOPS method

Sequential Multiobjective Problem Solving

$\mathbf{z} = (z1, z2, \ldots, zp)$

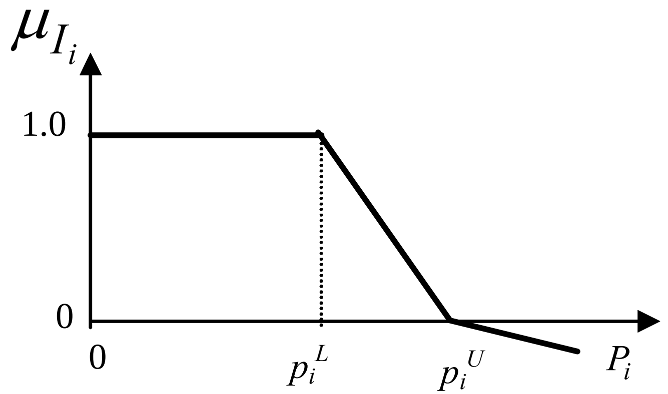
At most: $$z_i(\mathbf{x}) \leq AL_i; d_i = \frac{z_i(\mathbf{x})}{AL_i}$$

At least: $$z_i(\mathbf{x}) \geq AL_i; d_i = \frac{AL_i}{z_i(\mathbf{x})}$$

Surrogate objective function: $$s = \sum_{p=1}^{P} d_p$$

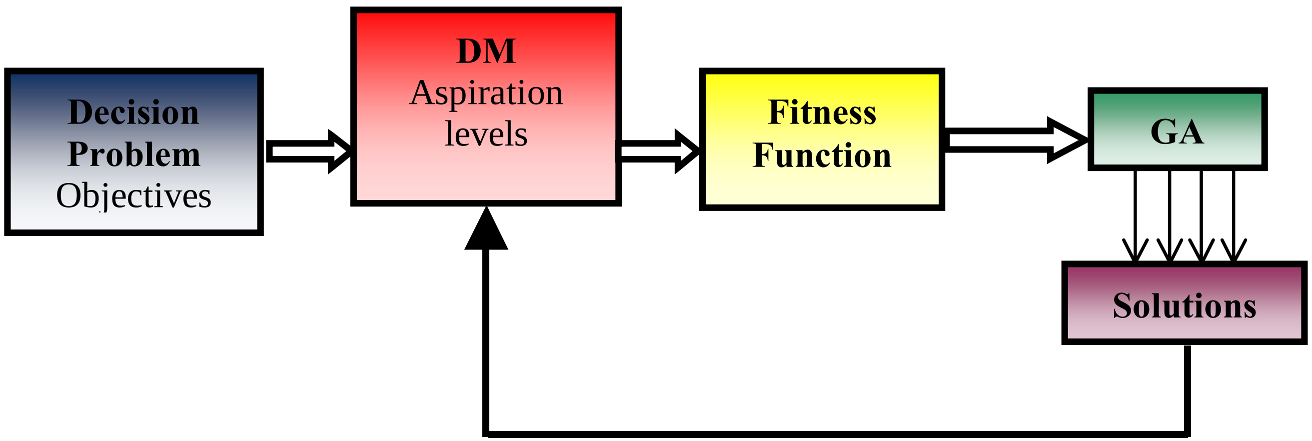
Objectives definition

Membership function of fuzzy individual schedule fitness:



$$\mu_{I_i}(P_i) = \begin{cases} 1 & \text{if} \quad P_i \le p_i^L \\ 1 - \dfrac{P_i - p_i^L}{p_i^U - p_i^L} & \text{if} \quad p_i^L < P_i < p_i^U \\ -0.5\dfrac{P_i - p_i^U}{p_i^U} & \text{if} \quad p_i^U \le P_i \end{cases}$$

Multi-objective genetic algorithm



Genetic Algorithm definition

- Binary representation
- Binary tournament selection
- Two-point crossover
- Four-point crossover
- Population size 960
- *T* number of generations

Experimental results

1000 generations, two-point crossover, Pmutation=0.1

SCHEDULE

	day 1	day 2	day 3	day 4	day 5	day 6	day 7
nurse 1	e	e	e	n	o	e	o
nurse 2	e	e	o	d	d	o	d
nurse 3	d	n	n	n	o	o	e
nurse 4	d	n	n	o	o	e	n
nurse 5	o	d	e	d	n	o	n
nurse 6	e	d	o	d	d	d	o
nurse 7	o	d	d	e	d	o	n
nurse 8	d	o	e	o	d	n	e

$d_1 = 1$; $d_2 = 0$; $d_3 = 0$;

Experimental results

1000 generations, four-point crossover, Pmutation=0.1

SCHEDULE

	day 1	day 2	day 3	day 4	day 5	day 6	day 7
nurse 1	e	o	d	n	e	d	o
nurse 2	e	e	o	n	o	e	n
nurse 3	e	e	o	o	e	e	e
nurse 4	o	d	e	n	o	n	e
nurse 5	n	e	o	d	o	e	e
nurse 6	o	e	d	d	e	o	d
nurse 7	d	o	e	o	e	d	d
nurse 8	d	o	n	e	n	o	n

$d_1 = 0.875$; $d_2 = 0$; $d_3 = 0$;

Conclusion and Future Work

- The approach proposed has two main characteristics:
  - The nurse scheduling problem is defined as a muti-objective problem with fuzzy individual schedule fitness
  - A hybrid approach based on an interactive sequential method combined with a genetic algorithms is developed
- The use of fuzzy sets is beneficial when subjective judgement is appropriate to use. Individual nurses preferences

Future Work

- The Head nurse's preferences can be modelled using fuzzy sets
- Linguistically quantified statements can be also used to prioritise the objectives
- Other constraints can be added to the problem

Nurse Scheduling Problem

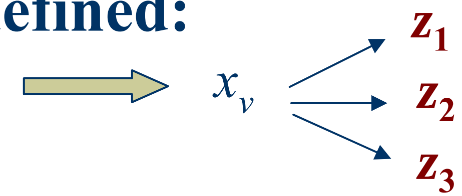
Table Nurses required in a day

	Nurse Grade *g*		
Shift	A	B	C
day-shift	1	1	2
evening-shift	0	2	0
night-shift	1	1	1

Fitness function definition

The following set and vectors are defined:

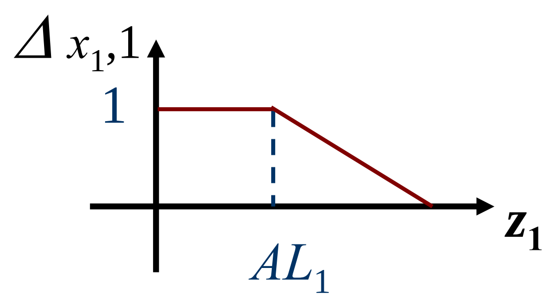
Candidate solutions $X=\{x_1, x_2, \ldots, x_V\}$



Aspirations levels $AL = [AL_1, AL_2, \ldots, AL_N]$ where N is the number of objectives.

Vector of distances between the aspiration levels and objective values

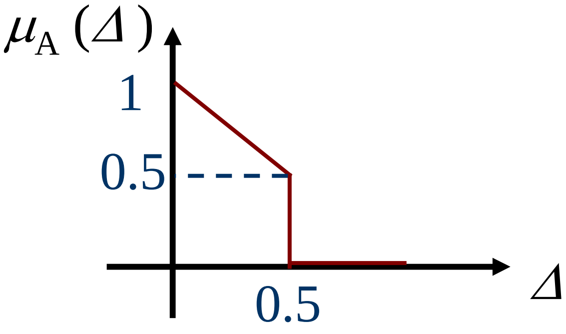
$$\Delta_{x_v} = [\Delta_{x_v,1}, \Delta_{x_v,2}, \ldots \Delta_{x_v,N}]$$



Fitness function definition

Fuzzy set A = acceptable distance between the objective value and the aspiration level

The membership function μ_A represents the degree to which an achieved objective value satisfies the DM with respect to its distance from the aspiration level AL_n.



Fitness function definition

linguistically quantified statement = "*most* distances between the achieved objective values and the aspiration levels are *acceptable*"

$$\text{"}Q\ \Delta_{x_v}\text{'s are } A\text{"}$$

Fitness function D is defined as:

$$D(x_v) = \text{Truth}[\text{"}Q\ \Delta_{x_v}\text{'s are } A\text{"}] = \mu_Q(r_{x_v}) \in [0, 1]$$

Fitness function definition

Yager's algebraic approach:

$$r_{x_v} = \frac{1}{N} \sum_{n=1}^{N} \mu_A(\Delta_{x_v,n})$$

For the quantifier "*most*":

$$\mu_Q(r_{x_v}) = e^{-50(r_{x_v} - 1)^2}$$

Fitness function definition

Find $x_v^* \in X$ that maximises the degree of truth of the linguistically quantified fitness function:

$$D(x_v^*) = \max_{x_v \in X} D(x_v)$$