Modelling the operation of a paediatric intensive care unit (PICU)

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Outline of talk

- Brief background
- PICU operation in theory and in practice
- Modelling bed demand variation for PICU
- Optimisation to assist resource and admission planning
- The future
Background
Great Ormond Street Hospital (GOSH)

1859

2006
Collaborators

- Martin Utley (CORU)
- Christina Pagel (CORU)
- Mary Gallivan (ex-Hammersmith)
- Tom Treasure (Guys)
- Mark Peters (PICU, Great Ormond St)
- + many other helpful staff at GOSH
Biographical note

Pure maths

Work Study

Traffic Engineering

Clinical OR
Some slides carry a health warning

Mathematical content

Apologies to non-mathematicians
Traffic systems - dynamic queue control

KEY PROBLEMS: Reserve Capacity Unpredictability Blocking
The conveyor belt model of capacity needs

Capacity required = Average daily number of admissions × Variability

Length of stay
A simple extension to the conveyor belt model

Booked admissions

Length of stay variable

DELIBERATELY SPARSE ASSUMPTIONS
Full attendance
No emergency admissions

How many beds are needed to honour commitments?
Variable length of stay causes variation in bed demand

How best to take account of case mix?

Booked admissions

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<tr>
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Attendance?

Emergency admissions?

Length of stay?

- Shorter stays
- Longer stays
More realistic but more complex analysis

\[ E(t_d) = \sum_{b=0}^{d} [n_{(d-b)}(1-v) + E(i_{(d-b)})]p_b \]

\[ Var(t_d) = \sum_{b=0}^{d} [(n_{(d-b)}(1-v) + E(i_{(d-b)})p_b(1-p_b) + p_b^2(N_{(d-b)}v(1-v)-Var(i_{(d-b)}))] \]

Computational complexity issues

Suppose 50 admissions to be planned. There are many different potential admission patterns. Suppose a millionth of a second to evaluate each option.

Q) How long to evaluate all possibilities and pick the best?

ANSWER

Longer than the age of the universe
Effects of cyclic admissions profiles

Decision variables: numbers of bookings

Mean bed demand

\[ \mu_d = \sum_{h=0}^{H} \sum_{c=1}^{C} n_{h,c} \sum_{w=0}^{\infty} p_c^{h,(WC + d - c)} \]

Variance of bed demand

\[ \sigma_d^2 = \sum_{h=1}^{H} \sum_{c=1}^{C} \sum_{w=0}^{\infty} n_{h,c} p_c^{h,(WC+d-c)} \left(1 - p_c^{h,(WC+d-j)}\right) \]

(Both cyclic)

\[ + \sum_{c=1}^{C} n_{0,c} \sum_{w=0}^{\infty} p_c^{h,(WC+d-c)} \]

Length of stay distributions

Goodness me, that’s linear

Gallivan S, Utley M, ‘Modelling admissions booking of elective in-patients into a treatment centre’

IMA J. Management Math, 16, 305-315, 2005
Mathematical optimisation to assist hospital operation

Input data

<table>
<thead>
<tr>
<th>Types of patient</th>
<th>Length of stay Distributions</th>
<th>Emergency Admission Rates</th>
<th>“Did not attend” rates</th>
<th>Contractual obligations</th>
</tr>
</thead>
</table>

Optimum admissions plan
Weekly pattern of bed needs
Maximised reserve capacity

Reserve capacity (%)

Optimised admissions

WARNING HARD SUMS
PICU operation in theory and in practice
Start of project at Great Ormond Street

Do you think we could fund a student to investigate optimised admissions planning for PICU?
Similar mathematical analysis . . . but

- GOSH is a tertiary referral centre
- Need to find out what happens in PICU
- Gather data from routine hospital sources
- Need “low tech” computer implementation
- Many emergency admissions come from Children’s Acute Transfer Service (CATS)
- Need to find out how CATS works
Elective PICU admissions “in theory”

Booked admissions

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Attendance?

Emergency admissions?

Length of stay?
Elective PICU admissions in “practice”

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Attendance?

Emergency admissions?

Length of stay?
Children’s acute transfer service (CATS)

- A centralised paediatric emergency service served by several PICUs
- Provides telephone advice and triaging
- Transfers very sick children to a PICU
- PICU receives transfer according to a set of rules:

  Referrer can state preference for which PICU
  
  OR, destination PICU determined by a rota
  
  BUT, PICU on rota may be full . . . etc
Emergency PICU admissions under the microscope

Booked admissions

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Attendance?

Emergency admissions?

Emergency admissions

Some internal, some external
LOS heterogeneous
CATs admissions not random

Length of stay?
Patient groupings according to length of stay in PICU (LOS)

<table>
<thead>
<tr>
<th>Grouping</th>
<th>Median Length of Stay</th>
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<tbody>
<tr>
<td>Respiratory emergency</td>
<td>3.9 days</td>
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<tr>
<td>Or Other emergency with previous ICU stay</td>
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</tr>
<tr>
<td>Respiratory booked</td>
<td>2.0 days</td>
</tr>
<tr>
<td>Or Other emergency with no previous ICU stay</td>
<td></td>
</tr>
<tr>
<td>Or Post-operative booked with previous ICU stay</td>
<td></td>
</tr>
<tr>
<td>Other booked</td>
<td>1.0 days</td>
</tr>
<tr>
<td>Or Post-operative emergency</td>
<td></td>
</tr>
<tr>
<td>Or Post-operative booked with no previous ICU stay</td>
<td></td>
</tr>
</tbody>
</table>
Disaggregating patients according to length of PICU stay

Proportion still resident (%)

Time since admitted to PICU – Log scale
Modelling bed demand variation for PICU
Example of variation in demand for PICU beds

Proportion of days

Number of patients in PICU at some time during a Monday
Disaggregated demand

Planned admission days

Elective

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Unplanned admissions

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Variable LOS

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Stochastic formulae for PICU bed demand

Mean bed demand

\[
\mu_d = \sum_{h=0}^{H} \sum_{c=1}^{C} n_{h,c} \sum_{w=0}^{\infty} p_{c,(wC+d-c)}^{h}
\]

Variance of bed demand

\[
\sigma_d^2 = \sum_{h=1}^{H} \sum_{c=1}^{C} n_{h,c} \sum_{w=0}^{\infty} p_{c,(wC+d-c)}^{h} \left(1 - p_{c,(wC+d-j)}^{h}\right)
\]

\[
+ \sum_{c=1}^{C} n_{0,c} \sum_{w=0}^{\infty} p_{c,(wC+d-c)}^{h}
\]

Length of stay distributions

numbers of expected admissions

WARNING HARD SUMS
Stochastic analysis

Planned admission days

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Unplanned admissions

Mean and variance of bed demand by day of week

WARNING HARD SUMS
Optimisation to assist resource and admission planning
Optimisation analysis

Planned admission days

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<th>Elective</th>
<th>Emergency</th>
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Unplanned admissions

Mean and variance of bed demand by day of week

DECISION VARIABLES
Analysis implemented in simple EXCEL system

Paediatric
Intensive Care
Analysis using Stochastic System Optimisation

Inventing acronyms: Waste Of Money Brains And Time
Optimisation smooths out demand over the week

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<tr>
<th>Reserve Capacity (%)</th>
<th>Upper 95% limit of bed demand</th>
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<tbody>
<tr>
<td>35.53</td>
<td>13.93</td>
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<tr>
<td>34.41</td>
<td>16.45</td>
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<tr>
<td>34.53</td>
<td>15.51</td>
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<tr>
<td>34.92</td>
<td>14.88</td>
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<tr>
<td>33.76</td>
<td>14.73</td>
</tr>
<tr>
<td>34.45</td>
<td>14.42</td>
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<tr>
<td>34.92</td>
<td>14.17</td>
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<table>
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<tr>
<th>Mean bed demand each day</th>
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<tr>
<td>Stream 1</td>
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<tr>
<td>-----------</td>
</tr>
<tr>
<td>Monday</td>
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The future

- The model works in theory, but will it work in practice?
- What is its role in reconfiguring PICU service?
- Can it be used in context of devising the CATS rota?
- Can one use such methods to schedule CATS staff shifts?