Using Markov models to help us better understand the Relationship between Hospital & Community Care

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The Patient Care Process

- Patients progress through stages of care, e.g. acute, diagnosis, treatment, rehabilitation and long-stay.
- Most patients are eventually treated, rehabilitated and discharged.
- The small proportion who become long-stay may remain in hospital for years.
- These patients may be very consuming of resources and thereby distort the statistics and cost implications.
Patient Flows

Acute Care

Conversion rate \( v \)

Long-Stay

Discharge / Death rate \( r \)

Discharge / Death rate \( d \)

Admission
Phase-type Distributions

- Phase-type distributions describe the time to absorption of a finite Markov chain in continuous time, when there is a single absorbing state and the stochastic process starts in a transient state.

- Distributions of this form have considerable generality, and include exponential (single phase), Erlang and mixed exponential distributions.

- Indeed, any continuous distribution with non-negative support can be arbitrarily closely approximated by one of phase-type form.
The Data

- The data analysed refer to male geriatric patients at St George's Hospital, London, over the period 1969-85.
- Durations of hospital treatment were available from a number of patients, along with two covariates: age at admission and year of admission.
- The analysis was initially concerned with finding a suitable distribution to describe the variation in the duration times and assessing the effects of the covariates on this distribution.
The Coxian Phase Type Distribution

Admission to hospital → Phase 1 → Phase 2 → ... → Phase n

λ_1, μ_1, λ_2, μ_2, ..., λ_{n-1}, μ_n

Discharged from hospital
The Probability Density Function

The probability density function of the random time $T$ taken for the process to be absorbed into state $n+1$ is then given by:

$$f(t) = p \exp\{Qt\}q$$

where $p = (1 \ 0 \ 0 \ ... \ 0 \ 0)$, $q = (\mu_1 \ \mu_2 \ ... \ \mu_n)^T$ and the matrix $Q$ is:

$$Q = 
\begin{pmatrix}
-(\lambda_1 + \mu_1) & \lambda_1 & 0 & \ldots & 0 & 0 \\
0 & -(\lambda_2 + \mu_2) & \lambda_2 & \ldots & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & \ldots & -(\lambda_{n-1} + \mu_{n-1}) & \lambda_{n-1} \\
0 & 0 & 0 & \ldots & 0 & -\mu_n
\end{pmatrix}$$
The four phase distribution

Figure 1: observed (_) and fitted phase-type (---) distributions
Adding Covariates to the model

- Covariates were available on age of each patient at admission and year of admission,
- they were incorporated into the phase-type distribution by allowing the parameters to depend on them through log-linear functions
- Of interest were the differential effects of the two covariates,
  - with year of admission only significantly affecting the phase 1 and 2 parameters, (2 x log-likelihood ratio = 13.83 on 3 d.f.)
  - age at admission only significantly affecting the phase 3 parameters (2 x log-likelihood ratio = 24.17 on 2 d.f.),
  - and neither covariate significantly affecting phase 4 parameters.
Extending the Model to Hospital and Community Care

- Geriatric patients typically have a number of spells in hospital, interspersed with spells in the community.
- We may therefore also use a phase-type distribution to describe spells in the community.
- In addition, patients may have different events that terminate a period of care:
  - For hospital patients these were discharge, transfer (to another hospital) and death
  - For community care these were admission to hospital or death
Combined Hospital and Community Care
Results for Combined Hospital and Community Care Model

- Over 75% of patients went through phases 1 and 2 of hospital care.
- Those patients who went into phases 3 and 4 have most opportunities for rehabilitation and stay longer in hospital care, particularly the younger patients.
- In the community, younger patients fared better than older patients.
- The greatest potential for improvement is to concentrate resources on the rehabilitation of younger patients.
We define $m$ additional community phases, with $\alpha_1, \ldots, \alpha_m$ describing sequential transitions between these community phases and $\beta_1, \ldots, \beta_m$ describing transitions from phases to phase $n+m+1$ (death).

In addition we represent transitions between hospital state $i$ and the community care phase 1 by $\nu_i: i=1, \ldots, n$ and transitions between community state $i$ and the hospital phase 1 by $\gamma_i: i=1, \ldots, m$. 

Notation for Modelling the System
Representing the Markov system

The whole system is then represented by the matrix $Q$:

\[
Q = \begin{pmatrix}
-(\lambda_1 + \mu_1 + \nu_1) & \lambda_1 & 0 & 0 \\
0 & -(\lambda_2 + \mu_2 + \nu_2) & \lambda_2 & 0 \\
. & . & . & . \\
0 & 0 & -(\mu_1 + \nu_1) & \nu_1 \\
\gamma_1 & 0 & 0 & -(\lambda_1 + \beta_1 + \gamma_1) \\
\gamma_2 & 0 & 0 & 0 \\
. & . & . & . \\
\gamma_n & 0 & 0 & 0 \\
\end{pmatrix}
\]

and the transition matrix by:

\[
A = \begin{pmatrix}
0 & \frac{\lambda_1}{\lambda_1 + \mu_1 + \nu_1} & 0 & 0 \\
0 & 0 & \frac{\lambda_2}{\lambda_2 + \mu_2 + \nu_2} & 0 \\
. & . & . & . \\
0 & 0 & 0 & \frac{\nu_1}{\lambda_2 + \mu_2 + \nu_2} \\
\gamma_1 & 0 & 0 & -\frac{\alpha_1}{\alpha_1 + \beta_1 + \gamma_1} \\
\gamma_2 & 0 & 0 & 0 \\
. & . & . & . \\
\gamma_n & 0 & 0 & 0 \\
\end{pmatrix}
\]
Number of entries to each state for the 3 state model

The matrix of expected number of entries to each state, before death is:

\[
\begin{pmatrix}
1 & \frac{\lambda_1}{\lambda_i + \mu_i + v_1} & \frac{\lambda_1}{\lambda_i + \mu_i + v_1} + \frac{v_i}{\lambda_i + \mu_i + v_1} \\
\frac{v_2}{\mu_2 + v_2} \cdot \left( \frac{\gamma_i}{\gamma_i + \beta_i} \right) & \frac{\lambda_i}{\lambda_i + \mu_i + v_1} \cdot \left( \frac{v_2}{\mu_2 + v_2} \right) & \frac{v_2}{\mu_2 + v_2} \\
\frac{\gamma_i}{\gamma_i + \beta_i} & \frac{\lambda_i}{\lambda_i + \mu_i + v_1} & 1 \\
\end{pmatrix}
\]

where:

\[
n_{11} = (1 - \frac{\lambda_i}{\lambda_i + \mu_i + v_1} \cdot \frac{v_2}{\mu_2 + v_2} \cdot \frac{\gamma_i}{\gamma_i + \beta_i} - \frac{v_1}{\lambda_i + \mu_i + v_1} \cdot \frac{\gamma_i}{\gamma_i + \beta_i})^{-1}
\]

\[
n_{12} = n_{11} \cdot \frac{\lambda_i}{\lambda_i + \mu_i + v_1} \cdot \frac{v_2}{\mu_2 + v_2} + \frac{v_1}{\lambda_i + \mu_i + v_1}
\]

and \( \Delta \) is the determinant of the transition matrix.
Using the Markov model of the whole system

- Using a phase type representation for transition inside the hospital and community components respectively we develop a Markov model of the whole system and use it to quantify performance metrics, such as:
  - Mean and variance of number of admissions to hospital before death
  - Mean and variance of total time from first admission to death spent in hospital and in the community.

- We illustrate these results using data from a London hospital for a 3 and 7 state model respectively.
Results for the three state model

<table>
<thead>
<tr>
<th>Mean admissions to hospital</th>
<th>Variance of admissions to hospital</th>
<th>Mean number of discharges to the community</th>
<th>Variance of discharges to the community</th>
<th>Mean total time in hospital (days)</th>
<th>Mean total time in the community (days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.28</td>
<td>0.35</td>
<td>0.50</td>
<td>0.53</td>
<td>65.19</td>
<td>287.19</td>
</tr>
</tbody>
</table>
The RIGHT project

- RIGHT ((Research Into Global Healthcare Tools) is EPSRC funded project with partners Brunel University, University of Southampton, Cambridge University and University of Ulster.
- It aims to classify the needs of decision-makers as they set about designing their service provision, and to match those needs to the nature of the result they want and to their resources.
- It will develop a framework that connects these user needs to user resources.
- RIGHT is a 2-year feasibility programme taking a pragmatic approach, with a tool-kit as a hard output. It aims to work with a variety of stakeholders who have defined a set of initial exemplars.
The problem

Can we connect up all these elements and feed information between the layers?
RIGHT at UU

- HPSSNI have awarded additional funding to support a post-doc for 3 years to collect data on stroke patients from the combined health and social services in Northern Ireland.
- A PhD student (Lalit Garg) is extending previous models of integrated hospital and community care that have been developed in collaboration with Professor Peter Millard.
- The models will incorporate aspects of decision nodes, costing and solutions using different technologies.
- The methods developed will constitute an exemplar within RIGHT and will be evaluated using the RIGHT Framework.
Conclusion

- The use of Markov models enables a distribution with the right sort of features to be used in analysing patient lengths of stay.
- The models may be used to provide results concerning the behaviour of patients in different phases of hospital or community care.
- Extending the approach to a Markov model of the whole patient care system can facilitate a holistic approach to patient care.