Queuing theory models: Qingcheng’s work lives on

In November 2003, Qingcheng Wang, a PhD student at the University of Westminster, died. Thanks to Thierry Chaussalet and Haifeng Xie his research aim lives on.


Qingcheng joined HSCMG in September 2002. His Masters degree was in mathematics and statistics and his long term goal was to become a Professor of Mathematics in China. Here he was investigating the use of a queuing networks approach to the study of patient flow in health care systems.

By modelling a health care system as a queuing network, we can tap into a wealth of well established methods that deal with this type of model. More importantly, queuing network models offer a natural platform to incorporate bed capacity constraints into the model, which will allow hospital managers to study how changes made to the system would affect performance in the long term, or in the mathematical terminology, when the system reaches equilibrium.

I remember, as if it was yesterday, asking Qingcheng “How long does it take for the system to reach equilibrium once the changes have taken place?” “Immediately” he replied, quick as a flash, turning to his computer keyboard to show me. Bearing in mind the Chinese saying – “Tell someone and they may forget; Teach them and they may remember; Involve them and they never forget” – I lay on the floor beside him until he realised that we were dealing with people’s lives. “Oh. I can do that” he said. But sadly he died before he had the chance to realise his ideas.

Qingcheng’s father, a master of ancient Chinese writing, gave me the drawing when he came to England. Now it graces our home, as a daily reminder of the contribution of Eastern and Western thought and mathematics to nosokinetics and to medicine.

An outline of the paper based on Qingcheng’s research concept is in page 5 of this issue. Pages 2-3 consider the potential benefit of On Line Analytical Programming (OLAP) in analysing hospital data. Page 4 reports the contribution mathematics is making to medicine in Melbourne.

Good news: Thanks to Mark Mackay, and the contributors, seven papers from the 2006 Nosokinetics meeting in Adelaide are in the February 2007 issue of the Australian Health Review.
On-Line Analytical Processing (OLAP) in the provision of health care: building capacity for exploratory data analysis and facilitating decision making. Christos Vasilakis, PhD

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OLAP can provide health managers and care givers with the capacity to perform fast interactive data exploration and analysis on large amounts of administrative and clinical data through an intuitive graphical user interface.

Health data have unique characteristics that distinguish them from common business examples, an aspect that makes the direct adaptation of the already established business oriented solutions difficult. Practical solutions and a working prototype software tool have been described and developed to aid in the analysis of bed occupancy and hospital length of stay (LOS).

The working prototype allows users to quickly generate graphs and tables with for example, daily admissions, discharges and bed occupancy over a period of time; moving averages of the above metrics over a period of time; and frequency distributions of hospital LOS. Here we illustrate the potential benefits to be gained by introducing OLAP data analysis using stroke data from the English National Hospital Episode Statistics (HES) database. The data concerns patients aged 65 and over discharged between April 1st and March 31st 1995: (148,251 Finished Consultant Episodes and 105,765 completed inpatient spells).

Figure 1 shows that the distribution of the LOS ranges between weeks and years. The mean LOS is 14.3 days, the median 7 days and the standard deviation 52 days, giving a coefficient of variation of 363%. The positive difference between the mean and the median suggests that the data is skewed, with a long tail of distribution to the right.

Most stroke patients are short stay. The 25th percentile is 3 days and the 75th 15 days, however, as the strata plot below shows, the majority of the beds (almost 90%) are constantly occupied by patients who stay for more than one week, Figure 2.

The strata plot shows the pattern of daily bed occupancy between April 1994 and Feb 1995. Notice the saw tooth nature of weekly bed occupancy and the ‘shock’ of Christmas. See how the short stay bed numbers decrease as patients staying for three weeks or less are discharged, and the longer stay patients increase as social services take their annual break.

Notice too, the artefact of decreasing long stay patients, because the downloaded data did not include a bed census on the last day.
Using an OLAP software tool hospital managers and care givers can easily bring into or remove from the graphs and tables the dimensions and metrics contained in the data. For example, Figure 3 shows how destination at discharge influences the average LOS of patients aged 65 and over discharged from English acute hospitals.

![Figure 3. Average LOS per discharge destination of stroke patients aged 65+](image)

Dimensions may include patient demographic information (e.g. patient sex, age), clinical information (e.g. diagnosis, treatment) or administrative information (e.g. source of admission, discharge destination). Metrics include number of patients in the hospital, number of admissions and discharges (total or daily numbers), hospital bed days, hospital LOS (average and variance), daily occupancy (crude and moving average).

In Figure 4 the number of discharges, the mean (▲) and standard deviation (♦) of the LOS of stroke patients aged 65 and over in the fourteen regional health authorities in England is displayed.

![Figure 4. Number of spells, average and standard deviation of LOS of stroke patients aged 65+](image)

Clearly, there are marked differences in the numbers treated and in the statistical values of the LOS distribution. A possible explanation is differences in the local availability of resources for transfer elsewhere.

The research described here was part of a doctoral thesis. The prototype has been tested on both hospital administrative databases and national health databases (Hospital Episode Statistics). It has been developed using widely available software including MS Access, MS Excel, and SQL Server 2000 Analysis Services, while the full OLAP functionality can also be accessed by using a web browser.

References
Research in Action: La Trobe University & Bendigo Health, Melbourne

**Maths is good for your health**  
*(Reprinted with permission, La Trobe University Bulletin, Oct 2006, p. 10)*

Hospitals need the skills of doctors, nurses, psychologists, physiotherapists, dieticians and technicians to ensure the health of patients. However, they cannot maximise patient health outcomes unless they also make use of the skills of mathematicians.

This is the view of Professor Terry Mills from the Department of Mathematics and Statistics at the Bendigo campus, a specialist in applying mathematics and statistics in health care.

Professor Mills, who has been teaching mathematics at the Bendigo campus since 1975, has worked as Honorary Senior Research Fellow at Bendigo Health since 1998. He says many problems in health care lend themselves to quantitative analysis.

‘We analyse data using methods from multivariate statistical analysis, and machine learning methods from computer science. We apply advanced quantitative ideas to practical problems in hospitals. ‘We also use our skills in these fields – and this has never before been applied to health care – to analyse the results of patient surveys.’

One major field concerns patient satisfaction. ‘Today there is a strong focus on patient-centred care. An important indicator in health care is the level of patient satisfaction, and this gives rise to the question as to what are the factors that distinguish between hospital patients who are, and are not satisfied?’

‘Patient satisfaction has a bearing on health care outcomes. Satisfied patients feel they have received value for money and time spent and, from a psychological aspect, feel better. This has a practical bearing on the result of health care because satisfied patients tend to obey advice from health carers about taking medicines, diet and exercise,’ Professor Mills says. Another project aims to forecast the demand for emergency services in Bendigo.

‘More than 30,000 patients go through Bendigo Health’s emergency department each year. If we can forecast not only the numbers, types of injury, and the busiest periods, it would greatly assist the rostering of staff. ‘We hope to have the result of this project by the end of the year. We are also looking at patient flow in general. To forecast this accurately is important because Bendigo Health is planning a new unit that offers transition care.

‘This will help elderly people make a transition from hospital to home or other accommodation. In planning this facility, a key question is: How many places will be required in this new unit?’ One answer can be obtained using mathematical modelling.

‘Another project is to determine how we can use mathematics to describe how patients flow through a hospital. This is part of a new branch of knowledge known as nosokinetics’.” These projects are multidisciplinary and require the expertise of researchers, both clinicians and academics, from different professions at La Trobe and Bendigo Health.

Professor Mills gave papers on results from his research at the ‘International Conference on Health and Social Care Modelling and Applications’ in Adelaide and the ‘Better State of Hospitals Conference’ in Melbourne, this year.
A Closed Queueing Network Approach to the Analysis of Patient Flow in Health Care Systems.

When planning bed allocation to hospital specialties, there is a clear distinction between services with constrained and unconstrained bed allocations. General medicine is a front-line, medical emergency service. ‘Bed borrowing’ by admitting physicians shows that their bed allocation is unconstrained. Practically, therefore, general medicine has as many beds (or trolleys) as required at any time. In contrast, geriatric medicine is a secondary, community supportive, service with a fixed number of beds. And waiting lists for admission to geriatric medical beds indicate that the bed allocation is constrained. In mathematical modelling terms, a department with a fixed number of beds and operating at full capacity is said to be a “closed” system.

Queueing network models consist of a number of server nodes each consisting of a number of servers, whose property is typically defined by the service rate (i.e. the number of requests that each server can deal with in a unit time), and the corresponding service time (i.e. the time needed for a server to finish a request), which is assumed to follow an (negative) exponential distribution.

In a health care system (for example, a geriatric department) patients can be regarded as progressing through a set of logical stages that resemble acute care, rehabilitation and long-stay care. We therefore treat a service node as one of the stages (or phases) that a patient progresses through the system. The servers at a service node are all the hospital beds in the system performing the corresponding task at the time. For instance, in a ward round of a twenty-bed ward, a consultant might find that, there are twelve beds occupied by “phase 1” (e.g. acute care) patients, six by “phase 2” (e.g. rehabilitation) patients, and two by “phase 3” (e.g. long-stay) patients. In this case, at the time of the round, there are twelve servers at node 1, six at node 2, and two at node 3. However, these numbers will vary with time depending on the use of each bed as patients’ progress through care phases.

The novelty of this approach brings models that are commonly used in other fields (such as computer network and telecommunication system) into the modelling of health care systems. As a result, it enables us to have access to a range of established methods that deal with queuing network models. More importantly, this approach provides a natural platform for incorporating a capacity constraint on the system (e.g. the total number of beds available in a department). By varying different model parameters such as bed allocation, average length-of-stay in each phase, and transfer probability between phases, clinicians and health care system managers can study “what-if” questions related to the long-term effect of changes to the current policies.

The paper demonstrates this modelling approach in the case of a geriatric department using parameters from a North London geriatric medical service. Assuming that staff behaviour does not change, our findings support an early observation by Struthers (BMJ 1963,1:470) that improving the treatment of chronic patients is, in the long term, the best way to reduce bed blocking and to improve throughput in the department.

Although the closed system described in this paper is reasonable for health care systems with constrained bed allocations and waiting lists for admission (such as geriatric medicine) its applicability in other specialities may be limited. Further work is ongoing to extend this approach to “semi-open” systems, i.e. a department with a constrained bed capacity which will remain open if there is an empty bed available, and is closed if full capacity is reached – which provides a more realistic representation to the running of a department.

What responsibility do we have for the implementation of the models we construct? Based on problem structuring methods, Córdoba argues that ethics needs to be understood as a continuous development by individuals in relation to existing frameworks and codes. Using Foucault’s ideas, to enhance critical reflection about ethics, two main areas of inquiry are suggested, namely, individualisation of ethics and possibilities and constraints of ethics in power relations. There are no simple answers, but, if you are interested in the ethical values of modellers, this paper is worthy of consideration.


Research with the Social Services Department of the London Borough of Merton into survival time in residential and nursing care has developed a prototype software tool, with a user friendly graphical interface, that combines unit costs of care with an estimated underlying survival model to determine publicly funded residents. The underlying model forecasts the cost of known commitments in long-term care, i.e. how much of next years budget allocation must be set aside to pay for current residents. Feedback from care planners shows the tool helps of understanding of survival time and facilitates budget planning.


Commenting on the payment by results component of the new NHS plan, the authors argue that outcome, not activity, should be the ultimate validation of the effectiveness and quality of care. Taking the analogy of a train journey, you would expect to arrive on time and safely in return for your fare. They conclude by asking ‘Why then, when clinicians strive to deliver evidence based medicine, is the Department of Health introducing evidence-free policies.


Study in an urban emergency department in Ipswich hospital, Queensland, suggests that the National Emergency Department Overcrowding Study (NEDOCS) tool does not reflect the subjective assessment of senior staff. Developed in America, NEDOCS may not be useful in Australia. However, limitations are a single study, short term (three weeks) and no serious overcrowding during that time period.


The model is generic and treats both demand (patient arrivals) and services (beds) as being homogenous. It is not computationally intensive and gives insight into the problem of matching increasing demand with reducing resources. However, there are limitations as to its use, as the polynomial bounded network structure is based on normal distributions.


Process mapping into value streams are a key component of ‘Lean thinking’. Flinders hospital emergency department had been under pressure for several years. Many established methods had been tried. Three sessions with staff revealed two value streams - patients needing admission (40%) and those that could go home after treatment. Introducing two teams working to these goals minimized queues, decreased the service time and reduced the number of people leaving.

Health expenditure Australia 2004-05

Health expenditure Australia 2004-05 examines expenditure on health goods and services in Australia for 1994-95 to 2004-05. Australia spent over $87 billion on health in 2004-05, an estimated rise of $8 billion since 2003-04. Expenditure estimates by area of health expenditure, as a proportion of gross domestic product (GDP), on a per person basis, by state and territory, by comparison with selected OECD and Asia-Pacific countries, and by source of funding (Australian Government, other governments and the non-government sector) are presented. This report will be helpful to anyone interested in studying, analysing and comparing estimates of health expenditure in Australia.

Forthcoming conferences

5th IMA QUANTITATIVE MODELLING IN THE MANAGEMENT OF HEALTH CARE
Goodenough College, Central London on 2nd - 4th April 2007
Conference website / or the IMA website http://www.ima.org.uk/
The deadline for abstracts, 300 to 500 words has been extended to 20th January 2007. Authors who submitted to previous deadline will be notified in January. Abstracts to be mailed to Lucy.Nye@ima.org.uk.
Selected papers presented (oral or poster) will be published in the Springer journal Health Care Management Science or in IMA Journal of Mathematics.

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Young OR 15. University of Bath, Bath, UK 28th-30th March 2007
Health Stream Adele Marshall and Barry Shaw, Queen's University of Belfast

The Health Stream welcomes recent applications of Operational Research to healthcare decision support and health policy. Such topics include, but are not limited to, areas such as disease modelling and cost-effectiveness, medical decision-making, planning health services and logistics to include the modelling of health care interventions and patient care, resource allocation and workforce planning, performance measurement and evaluation, and clinical monitoring.

12th International Conference on Applied Stochastic Models and Data Analysis May 29 - June 1, 2007

20th IEEE International Symposium on COMPUTER-BASED MEDICAL SYSTEMS

Nosokinetics News: Three years old and growing strong
Professor J. Z. Young, professor of anatomy at University College Hospital, London taught us that “We grow oldest, when we are youngest”. If you don’t see an infant for a few months you notice the difference. Teenagers change but slowly and adults only really change in months or years if they are sick.

Three years ago we launched Nosokinetics News onto an unsuspecting world. Thanks to your support, from modest beginnings, we now mail to 437 people worldwide. Nosokinetics now has 299 Google hits on the web and 1,917 on my computer. But success has brought its problems and we need to change the way the newsletter is constructed and mailed.

This is the last newsletter that will be sent with personalised letters. Future newsletters will be sent using JISCmail, the UK academic mailing list. The benefit being simplicity, and ease to subscribe and unsubscribe. Everyone who is now on the mailing list will be automatically included in the new system.

Instructions about how others can register, and how to subscribe and unsubscribe will be sent in the new year. We intend to bring news about the research of different groups, to include a PhD research section in addition to short articles.

Feedback, contributions and leads to papers will always be welcome. These should be sent to nosokinetics@tiscali.co.uk. At this stage, looking forward to the 3rd birthday celebration – candles and cake – we are growing slowly but surely in our mission to bring a scientific approach to the planning and monitoring of health and social care. For the benefit of all.

Peter Millard : Editor Nosokinetics News