EMOTIONS ARE IMPORTANT IN MODELLING

Mathematical modelling: how and why

By Gary Harrison, Simone Ivatts and Peter Millard

We welcome Professor Annabelle Mark’s constructive criticism of our goals and dreams (Ivatts & Millard A and B 2002). In responding here, we have recruited Professor Gary Harrison’s assistance to explain why our ‘gold standard’ remains the development of a mathematical model of patient flow.

Medicine advanced when pharmacologists developed compartmental models of flow. Likewise, hospital planning and patient care would advance faster if scientific models of flow complemented Professor Mark’s concern for patient emotions.

Much of her criticism comes from a misunderstanding of the proper limited use of a mathematical model (Mark 2003).

What is a model?
A model is a simplified caricature of a complex system, designed to answer a specific question. To understand the process of in-patient hospital care, managers need to understand how patients flow through hospital beds, which is the goal of our models.

Snowden’s Cynevin model is based on the paradox that knowledge is simultaneously a thing and a flow (Snowden 2002). The assumption behind the model is that “Humans, acting consciously, or unconsciously are capable of a collective imposition of order in their interactions that enables cause to be separated from effect and predictive and prescriptive models to be built” (Snowden 2002).

Feelings are important, and many staff working in hospitals are strained due to unremitting central pressure to change. Telling people to slow down when they are working flat out is clearly illogical (Mark 2003).

What if, however, our mathematical models showed them how to do so?

Rerailing the train
Mark derails our train because it ignored the passengers’ emotions (Mark 2003). Derailing the train does not help.

Mark seeks to assimilate the role of non-rational and emotional processes into decisions around demand management, but we seek to develop a new scientific approach to the planning of bed allocation and use.

The role of emotion
It is true that people run health services, and that emotions motivate people. Just as a nurse or doctor may feel great compassion for their patients and care deeply about helping them, but have to develop a certain professional detachment in order to make sound judgments about treatment, so a manager may care deeply about improving health care but must not let his personal feelings colour his professional judgment about the best ways to do so.

Models of emotions (Mark 2002) and models of flow may help both groups maintain emotional neutrality.

Model planes
A good model is formed to answer a spe-
cific question. To do so, the model must be simpler than the original but preserve some important properties of the original.

This principle can be illustrated by thinking about model airplanes. A child’s model airplane may preserve the appearance of some real airplane, but not be able to fly, or it may have a motor and be able to fly but not look like any full-scale airplane. Rarely does a model airplane preserve both properties.

Other applications
Other model airplanes are not toys. An engineer may use a model airplane that preserves the shape in order to test the aerodynamic properties in a wind tunnel. He does not want his model to have an engine; that would just be a distraction. He simply wants to eliminate turbulence in the airflow.

When we buy airline tickets online, we may be given an opportunity to select our seats using a model of the airplane that shows where the seats are in relation to the windows, wings, doors, etc. It does not bother us that this model is only two-dimensional when a real airplane is three-dimensional. It contains all the relevant information for its purpose.

The most complicated model airplanes are the flight simulators used to train pilots, but even they are simpler than a real airplane. We wouldn’t want them to actually crash and burn when the trainee makes a mistake.

Each purpose requires a different model: emphasising some characteristics and ignoring others.

Modelling health and social care
The real challenge for the modeller is to determine what is relevant and what is not. We would not want a model that contained all the complexities of the National Health Service, because we would not understand the model any better than we do the NHS!

We share Mark’s concerns about the development of complex models of patient movement that seek to explain the interactions between many things (Mark 2003).

Our models are designed to help managers understand the time component of resource use.

Modelling acute, rehabilitative and long-stay care
- There are sixty-four different ways that a hospital system providing acute, rehabilitative and long stay care can be changed
- You can increase or decrease the beds (2 x 2 x 2 = 8) or slow down or speed up the rate of discharge from each sector (2 x 2 x 2 = 8), 8 x 8 = 64
- Not all of these 64 ways are beneficial
- Some give the illusion of success, but long-term failure
- Others give little early benefit, but long-term success

A lesson from history
During the 1980s and 1990s, the UK government artificially speeded up the rate of flow through hospital care by fostering an entrepreneurial boom in the development of private and voluntary residential and nursing homes (Millard & Lee 1997).

Cashing in on this boom, thousands of hospital beds were closed. Now the NHS has changed from a welfare state to a welfare market, and rehabilitation is being rediscovered (Foote & Stanners 2002).

Whether our ‘gold standard’ model could have prevented the running-down of hospital geriatric services is unclear, but, despite Marks’ protestations, we still think that such a model could teach other countries not to make the same mistake.

Mathematical models
- Model-building usually requires several iterations to arrive at a useful result
- Our models are new: we are just beginning the second cycle of refinement
- Even if the models do not yet have precise predictive ability, model building is valuable
- Model-building and validation forces us to search for the essential features that impact on the question being asked

Figure 1 (over) shows how mathematical
EMOTIONS ARE IMPORTANT IN MODELLING

‘A model by itself cannot improve health care services: that depends on the actions and decisions of many individuals, including the administrators, doctors, staff, and the patients themselves. Yet mathematical models can show us where to focus our efforts and what are the most sensitive parts of the system.’

models are developed, solved and refined.

Lessons learnt so far
The following are some of the things we have learned from modelling bed usage. This section is divided into compartmental models, factory models and behavioural models to highlight the similarities and differences between compartmental models of hospital and social care and pharmacological models.

1. Compartmental models
1.1 BED OCCUPANCY ANALYSIS
The average midnight bed occupancy is considered to be a bad measure because it combines and confuses occupancy and use (Yates 1982). Fortunately, mathematical models based on exponential analysis of hospital bed occupancy time in bed census data show how occupied beds are actually being used (Harrison 1994; Harrison & Millard 1991).

Because the distribution of occupancy time in hospitals, residential homes and nursing homes all follow a mixed exponential pattern, decision makers can also determine how these services are being used (Millard, Christodoulou et al. 2001).

FIGURE 1. THE MODELLING PROCESS
An iterative process is used to develop mathematical models. First, you have a real-world problem. Next, a conceptual model is developed. Then the conceptual model is translated into a mathematical model (simplifying as necessary), and (if possible) the mathematics is solved. Finally, the solution has to be interpreted and compared with data relating to the real world problem. Step-by-step, several iterations are needed to reach a useful result.

Real-world problem

Conceptual model

A solution

Mathematical model
Look at Table 1. Notice the differences in the fractions of people discharged each day from different services. If you ever wondered why Local Authorities do not rapidly find residential care places for hospital discharges, notice that a vacancy arises in residential homes every two days (502 beds x 1/1000 x 2 days = 1 bed).

1.2 MIXED EXPONENTIALS AND STREAMS OF FLOW

The models that Harrison developed consider the outcome of a cohort of patients admitted on the same day. If a fixed percentage of admitted patients were being discharged each day, a graph of the number of patients still in the hospital over time would follow a single declining exponential curve.

Yet medical care is not like this, for a proportion of patients admitted each day have highly complex medical, social problems and psychological (including emotional) problems. So it is highly unlikely that the process of inpatient medical care will follow a single exponential curve.

Most specialties can be visualised as containing two streams of patients

- Most patients leave in the first, ‘quick’ stream
- Patients with complex illnesses and a slower recovery stage enter into a separate slower stream of flow
- The number of complex patients retained decreases in a second, slower, exponential decline
- In some types of service, such as, geriatric medicine and psychiatry, there is even a third long term care phase with an even slower exponential decline
- Thus the distributions of the times that cohorts of in-patients occupy beds must be represented as a sum of two or three exponential terms, called a mixed exponential distribution
- The longer a patient has been in the hospital, the greater the patient’s expected additional stay (Harrison 2001)

1.3 THE AVERAGE LENGTH OF STAY AT DISCHARGE IS A BAD MEASURE

The finding that patients move through hospitals in two or three streams of flow with different rates is ubiquitous in all types of departments.

- Intensive care doctors measure time in hours and days
- General medicine, surgery and orthopaedics, measure time in days and weeks
- Psychiatry and geriatric departments measure time in weeks and months.
- Despite these differences, the general shape is the same

It is the exponential nature of flow through hospitals that explains why the average length of stay of cohorts of discharged patients is a bad measure of inpatient activity (Harrison 2001).

The data to evaluate the rates of flow (namely, censuses, admission and discharge dates) are already being recorded, although Mark’s point that the recorders need to understand their significance (Mark 2003) is well taken.

1.4 BOUND AND UNBOUND PATIENTS

Models of drug absorption, distribution, metabolism and excretion explain why some drugs are rapidly excreted while others have a different excretion pattern because they bind to the body. Similarly, patients with similar primary medical diagnosis, but different social circumstances or complicating other illness, remain longer in hospitals (Marshall, MClean et al. 2001; Marshall, MClean et al. 2002).

These patients are only a small frac-
1. Current status

2. Working harder

3. Working smarter

The hexagrams illustrate the choices that face clinicians and managers as they seek to improve hospital performance. The choice to do one thing is the choice not to do something else.

Two streams of patients are flowing through nearly all hospital specialties.

Hexagram 2 illustrates attempts to increase turnover by rapidly discharging recently-admitted patients.

Hexagram 3 shows that the same throughput would occur if the inpatient management of second stream patients were improved.

The concept that underpins the hexagrams comes from the six-line models in the I Ching (the Chinese Book of Changes).
steady flow of patients through hospitals.

2.1. LINKS
The ability to discharge the long-term patients is influenced not only by internal style of practice but also by external factors such as availability of social services in the local area.

An analysis of the 1994 Hospital Episode data found that the use by Health Districts of nursing homes as a discharge destination for elderly people with stroke illness varied from one in ten to one in a hundred (Millard 1998).

A mathematical model has been developed which shows the links between hospital health care and social services (Taylor, McClean et al. 2000). Further development of these models would be worthwhile.

2.2 WAITING FOR DISCHARGE
Internal queues form in hospital departments when patients who could be discharged are waiting for an opening for placement (El-Darzi, Vasilakis et al. 1998). Also, delays in discharge occur during the prolonged Christmas holidays contribute to the shortage of acute hospital beds that frequently occurs in January (Vasilakis & El-Darzi 2001).

2.3 TOO FEW BEDS TO MEET DEMAND
Queuing theory models show why hospitals that seek to achieve 100% bed occupancy cannot succeed. The models show that eighty-five percent average bed occupancy may be the right target (Bagust, Place et al. 1999; Gorunescu, McClean et al. 2002).

2.4 VARIABILITY
There is a pronounced weekly cycle in admission rates and discharge rates, which account for a large fraction of the variability in patient numbers. The weekly cycle and bank holiday effects are due to the non-biological (factory) nature of modern hospitals.

There are also seasonal effects (Mackay 2001). Variability makes it hard for hospital managers to plan their services. This variability is driven not only by variability in the demand for services, but also by variability in the discharge rates.

KEY POINTS
- Mathematical models are a scientific tool of studying the flow of patients and clients through health and social care systems
- A model is always simpler than the ‘real environment’, but preserves important characteristics needed to answer a specific question
- Plans based on single averages of bed occupancy, bed emptiness and average length of stay should be put into the dustbin of history
- Mixed exponentials distributions are a better way of understanding bed usage
- The distributions of occupancy times in different types of department have similar characteristics but different time scales
- Only a small fraction of the admissions become long-stay patients, but they use a much larger portion of resources
- Emotions are an important part of our everyday lives, but alone cannot explain or study the complexities of health care usage

3. Behavioural models
There are also some conjectures from our modelling exercise about the behaviour of medical staff that cannot be proven with our current models, or perhaps not with any model.

- Doctors adjust their referral and discharge behaviour to take account of local resources.
- Agents of change modify staff behaviour from one constant form to another constant form.
- Under pressure, staff adjust admissions, borrow beds, and create queues rather than adjust their learnt behaviour

Here the ideas of Annabelle Mark about personal ownership and responsibility come into play.

A model by itself cannot improve health care services; it depends on the actions and decisions of many individuals, including the administrators, doctors, staff, and the patients themselves.

Yet mathematical models can show us where to focus our efforts and what are the most sensitive parts of the system.

Conclusion
The National Health Service has many sick hospitals and thousands of sick people in need of care. Examined from without, hospitals have immense complexity; but within there is simplicity, for the date of admission of all patients is known.

Prescribing improved after pharmacologists developed models that showed how drugs were absorbed, distributed, metabolised and excreted. Similarly, the compartment flow models that we are building enable us to better understand the dynamic interactions between health and social care.

Thus our gold standard remains the creation, validation and practical implementation of models that will enable decision makers to better meet and understand how to optimise care services for frail, sick and dependent people in the 21st century.

References
Bagust, A. Place, M. & Posnett, J. W. 1999,
EMOTIONS ARE IMPORTANT IN MODELLING

‘the compartment flow models that we are building enable us to better understand the dynamic interactions between health and social care’


