Symptoms and signs are nature’s expression of defence against disease. Trolley waits; cancelled operations; hospital-acquired infections; blocked beds; delayed discharges etc. all are signs of ‘sick’ hospitals. Symptoms are parliamentary questions, public disquiet and complaints.

In factories, it is well recognised that the quality of the finished product decreases if the conveyor belt goes too fast. Yet, for the last twenty years and for the foreseeable future the basic premise is that hospital treatment can be forever sped up with impunity.

Now NHS performance tables (based on speed of treatment and death rates) are being published in national newspapers to ‘name-and-shame’ failing hospitals. Yet factors inside and outside hospitals influence speed of treatment.

For example, in English Health Authorities, there have been considerable differences in the outcome, discharged home and discharged to nursing home for patients admitted to hospital with stroke illness (Lee et al. 1998). Soon, articulate pensioners will want to know why?

Currently, a comment in the Department of Health’s performance measure on length of stay states, “keeping people in hospital longer than is necessary can be an indicator of poor performance (Department of Health 1999)”. So, the pressure is on to shorten length of stay, seemingly, irrespective of need and the outcome of the treatment.

Disease presents differently in older people, and often comes disguised as social problems. To an older person and their family, there is a considerable difference between the outcome discharge to nursing home without rehabilitation and safe return home following quality in-patient care. The population is ageing, and community care will fail if dependent older people with social problems which mask illness are denied access to 21st century acute hospitals.

In our previous article (Ivatts and Millard, 2002), we showed that the methods used to measure, compare, contrast and plan hospital in-patient activity are seriously flawed. We also argued that scientifically valid tools should be used to measure and model in-patient activity.

**Starting from here**

Our aim in writing these papers is to take the reader from Level Zero of Seymour’s hierarchy of possible levels of understanding of modelling methods to Level A, where a few general rules of thumb or ‘heuristics’ are understood due to a general reading of the modelling literature (Seymour, 2001). References in this paper guide readers to recent research papers that underpin the concepts that we are developing.

In Level B of Seymour’s hierarchy, understanding of clinicians’ (and, we would add, managers’) own data-set at one point in time is obtained, and simple “what-if” scenarios are constructed.

Information about the tools that we are using can also be obtained from the University of Westminster Health and Social Care Modelling Group web site, at http://www2.wmin.ac.uk/hscmg

In the first paper, we used a cube with black and white squares to illustrate the problem of “bed-blocking” (Figure 2,

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In the second of two articles, based on Professor Millard’s recent lecture course, we explore the concept of “flow”, i.e. how patients actually move through hospital beds. The use of mathematical models to describe flow is outlined, and examples are given of practical applications in health care management.
MODELLING DEMAND

paper 1). Figure 1, in this paper, further develops the modelling concept to show how simple algebra adds a further explanatory variable to explain ‘bed-blocking’.

Models simplify complex systems
Toy trains are not real trains, but they do enable us to understand how real trains work. Changes made in the layout of a toy train track do not change the real world, but sharp corners explain what happens if trains go too fast.

We used a twenty-four hour train in our first paper (Figure 1, paper 1) to make the problem of measuring, modelling and planning hospitals easier to understand. By using the analogy of a train circling a track at constant speed, we can remove the emotion associated with discussion of patient care and focus on the problem of service improvement in a different way.

The train circles the track at a constant speed: time cannot be speeded up. To improve the passenger carrying performance of the train (hospital) one has several choices. To:
- add more carriages (more beds),
- stop passengers getting on board (reduce admissions in the over 75’s)
- stop passengers staying overnight (more day patients)
- persuade passengers to get off the train early and continue their journey elsewhere (early discharge, intermediate care and transfer to social care homes).

The problem with all of the above methods of improving performance is that they take, as a given, that the clinical management of patients within the hospital is optimal and cannot be changed.

Yet Figure 1 in this paper shows that the key explanatory variable associated with ‘bed-blocking’ is a medical decision, taken within the hospital, that no further actions can be taken within the hospital to solve the patient’s problem.

Thousands of facts are collected in modern hospitals. The challenge is to find the minimum number of facts that are needed to determine how beds are actually used. Some of the necessary facts, like age, sex, specialty, diagnosis, admission and discharge dates and discharge destination are already collected. Others are not.

Figure 1
The figure shows why the key to solving ‘bed-blocking’ may lie within the hospital. Assuming that a hospital has no empty beds, on any day new patients can only enter if other patients leave.

Patients designated as ‘bed-blockers’ change their status from active intervention to passive compliance. The patient recognises their changed status because the consultants and their teams pass by; the therapists move on to more rewarding patients; and the nurses still tend, but they do no longer actively encourage self-help.

In modelling terms, the patient has crossed a catastrophe hypersurface, a local behaviour-related decision-making threshold over which the patient had no control. And everyone waits for someone else to solve the problem.

Algebraically, the stable state equation is Ac = Bv; i.e. the stable state depends on the conversion rate within the hospital being the same as the rate of discharge of ‘bed-blocking patients’.
When Ac > Bv the hospital will slowly have fewer beds to meet acute care needs. Vice versa, if the hospital staff could change their learnt behaviour such that Ac < Bv then over weeks and months, more beds become available for admissions.

Figure 2 shows part of the problem that needs to be solved.

Model building (Bartholomew et al. 1991)
Models are built in four stages.
- Model-building is the role of the scientist
- Model-solving involves mathematicians
- Model testing requires applied research
- Model implementation involves decision makers

In the decade since Harrison and Millard (Harrison and Millard, 1991) reported a mathematical solution to a two-compartment model of flow in departments of geriatric medicine, a world-wide group of mathematicians; decision scientists; computer experts; health business analysts; care providers and clinicians have been working together to validate, expand and test new methods of measuring and modelling health and social care systems.

A scientifically-valid model of the interactions between health and social care is the gold standard for which we must aim. Stages 1, 2 and 3 have already been achieved. The time is now right for stage 4 (operational planning) to begin.

Operational Planning (Feldstein, 1963)
Key features of operational planning are:
- An operational model of the problem
- Implementation of the model with statistical evidence
- Solution of the problem by finding the alternative policy, which maximises a particular equation in the model

Human activity systems (McLean & Bustard 1997)
Hospitals are human activity systems. In human activity systems, human beings undertake some activity to achieve a purpose, e.g. a factory or a university.

Professors McLean and Bustard from the School of Information and Software Engineering at the University of Ulster defined the properties of a full geriatric care system as:
- Ongoing purpose - care of the elderly
- Mechanism for control - government control
Sub-systems - hospital, nursing homes
Interacting sub-systems - patient flows
Wider system - National Health Service
Boundary - catchment area
Resources - budget
Expectation of continuity – hopefully.

An important point about systems is that they form hierarchies. The authors also define a typical system hierarchy of health care as:
Level 1: Health Authority / Region
Level 2: Hospital
Level 3: Department
Level 4: Ward

Key points concerning a systems approach are:
The aim is to optimise performance
The approach takes a broad view, embracing social and political aspects
Optimum parameters are needed for the design, construction and operation of the system
Control is dependent on the feed back of accurate information
Performance measures typically focus on productivity, responsiveness and utilisation
Performance indicators are frequently interdependent

Changes in one part of the system can have a detrimental effect on other parts.

Care must be taken when instituting change in human activity systems, because sub-systems interact horizontally as well as vertically. Activity in hospitals cannot be considered alone, because hospital and community services interact.

To improve patient care, hospitals need to slow down
Treatment in acute hospitals cannot forever be speeded up without deleterious consequences on patients, staff and carers.

To escape from the conveyor belt mentality, performance measures are needed that take into account the biological, psychological and social characteristics that determine a patient's length of stay.

In biological terms:
Patients have simple, difficult or complex problems
They are ill and likely to recover, or they are not
They can walk or they cannot
They are continent or they are not

In psychological terms:
They have good mental function, or not
They want to return home, or not
They co-operate with staff, or not, etc.

In social terms:
They can have good or bad home circumstances
Their partner (if any) can be alive or dead
Their neighbours may be supportive, or not
Their relatives may support a home discharge, or not
The early discharge (intermediate care) team may or may not have places.

Unless the interactions between hospital and community care are taken into account, we reach the absurd situation in which the hospitals that institutionalise sick people in nursing homes without any rehabilitation get praised.

Meanwhile, the hospitals that minimise the need for long-term care, by giving sick people time to recover, get punished.

Uncertainty (Marshall et al. 2001)
In medicine, there are no certainties: just interacting probabilities.

Bayesian Belief Networks model the causal relationship between variables, where one variable is viewed as having a direct influence on another.

Although age is an explanatory variable for increasing length of stay, age alone does not determine how long a patient will occupy a bed.

For example, a male patient who lives alone admitted with stroke illness is most likely to spend over 50 days in the department of geriatric medicine, irrespective of his actual age.

Marshall’s cited study used Bayesian Belief Networks (BBN’s) to analyse a database concerning 4,722 patients admitted to the department of geriatric medicine at St. George’s Hospital, London.

The analysis showed that personal details interacted with admission reasons to determine dependency, and these factors interacted to determine length of stay.

Using Bayesian Belief Networks based on local data, clinicians and hospital managers could gain knowledge of the...
MODELLING DEMAND

The creation of a scientifically-valid model that shows the interactions between health and social care is the gold standard for which we must aim.

Modelling hospital flow (Harrison, 2001)

Prof. Harrison, from the College of Charleston, South Carolina, US developed the mathematical solutions to the two compartment (Harrison and Millard, 1991) and three compartment (Harrison, 1994) models of flow through health and social care services.

The models that Harrison developed are analogous to the compartmental models that are used to measure the absorption, distribution, metabolism and excretion of drugs. The difference between pharmacological models and Harrison models is that pharmacological models analyse the clearance of drugs, whereas Harrison model analyse the pattern of accumulation of patients.

Figure 3 illustrates the concept of one, two and three streams of flow and shows the services that are associated with the different models of flow. The only service that we have examined so far, in data sets from different parts of the world, that did not have an exponential fit to its occupancy data is learning difficulties.

The benefit of using exponential models of the flow of inpatients through health and social care systems is that decision makers can better understand the process of care (McClean and Millard, 1993a, 1993b). Also the short and long term outcome of possible decisions can be tested.

In future years, as the biopsychosocial nature of hospital human activity systems becomes better understood, clinicians and managers will discuss length of stay in hospitals in terms of streams of flow and half-lives. Single averages of complex systems will be placed in the dustbin of history.

Mixed exponential distributions in bed occupancy data imply:

a) That the length of stay of current in-patients is much longer than the length of stay of discharged patients.
b) That the average stay of a group of discharged patients will not correctly indicate the resources that are used for rehabilitation and long-stay care.
c) That the expected additional days’ stay of an in-patient increases dramatically the longer they occupy a bed.

Practical application

Several researchers have been able to apply these models to practical problems in the health service. Examples are:

- Explaining the Christmas bed crisis (Vasilakis et al, 2001)
- Queuing models of emptiness and occupancy (Gorunescu et al, 2002)
- Modelling the total system of care (Millard et al, 2001)
- Bayesian Belief networks in predicting length of stay (Marshall et al, 2001)
- A mathematical solution to six compartment model of the hospital and community system of geriatric medical care can be found in (Taylor et al, 2000)

Papers describing newer methods of analysing hospital activity include:

- Bed usage by stroke patients (Lee et al, 1998)
- Use of surgical beds (Millard et al, 2000)
- Use of medical beds (Christodoulou and Millard, 2000)
- Models that incorporate discharge destination (Mackay and Millard, 1999)
- Practical experience of modelling (Mackay, 2001)

Conclusion

Mathematical models are only as good as the theory on which they are based and the accuracy of the statistical evidence that is used. Linear models are now used to plan health and social care services. These models are based on concepts of pressure and force (MacStravic, 1984). In future years behavioural flow models will be used instead.

Figure 3
Stream found in different services

Cardiac surgery
Rehabilitation
Hospital long stay
Nursing homes
Home helps

General surgery
General medicine
Obstetrics
Orthopaedics

Geriatric medicine
Psychiatric hospitals
District services

Flow modelling identifies the presence of the different streams of flow and measures the contribution that each stream within the system gives to the whole process of care. One, two or three streams of flow with different dimensions of time occur in services dependent on the nature of the patients being admitted, the characteristics of the patients who are admitted e.g., pregnancy can be normal or associated with complex problems that require hospitalisation, so the bed in the obstetric department are mainly occupied by longer term patients while the day to day workload is delivering babies.
Linear models have served us well. But, the *reductio ad absurdum* of linear thinking is that no beds will eventually be needed to treat patients. Clearly, that conclusion is absurd.

Mathematically, as well as clinically, a forgotten key component that controls availability of resources in all other parts of the system is the decision that a patient needs long-term care.

The creation of a scientifically-valid model that shows the interactions between health and social care is the gold standard for which we must aim. If such a model were created, decision-making should improve. Policy makers could predict the results of proposed changes without continually experimenting with the real world (McClean and Bustard, 1996).

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